When, Where, and How Much to Invest for Enhancing Transportation Network Performance: Insights to Augment Decision Making

Presented by:
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Motivation (1)

- State DOTs must allocate available budget to set of projects
- Divisions within DOT focus on specific MOE
  - Congestion
  - Consumer surplus
  - Safety
  - Air Quality
- Focus in one MOE may impact other
Motivation (2)

- Budget is often limited
- Empirical models do not work (lack of behavioral component)
- Need to consider user behavior
- Single year versus multi-year investment
- MOEs are often conflicting
Motivation (3)

• Key question remains

“When, where and how much do we need to invest such that overall goal of a transportation agency is satisfied”

“Would it be prudent to invest now or wait”
Background

• The problem is multi-objective
• Considers multiple players
  - Decision makers (leaders)
  - Users (followers)
• Stackelberg’s game
• Problems are often non-linear and difficult to solve
Purpose of Project

- Produce a method of prioritizing projects
  - Consider Total System Travel Time (TSTT) performance measure

- Allocate the budget to priority links

- Provide State DOTs with a decision making tool
Methodology

• Bi-Level Optimization

• Planners
  - Upper Level Problem (ULP)
  - Minimize Total System Travel Time (TSTT)

• Users
  - Lower Level Problem (LLP)
  - Traffic Assignment (UE)
Data Required

- Number of Links in the network
  - Capacity
  - Length
  - Free Flow Travel Time
  - Alpha and Beta parameters
  - Connecting Nodes

- O/D Matrix

- Budget
Formulation

Upper Level problem (ULP)

Objective Function:

Minimize \( T_{\text{STT}} = \sum_a x_a t_a(x_a, y_a) \)

Subject to:

\[ \sum_{a \in A} g_a(y_a) \leq B \]

\[ 0 \leq y_a \leq c_a : \forall a \in A \]

Where:

- \( T_{\text{STT}} \): Total System Travel Time
- \( x_a \): Flow for link \( a \)
- \( y_a \): Capacity expansion for link ‘a’ (nonnegative real value)
- \( t_a \): Travel time for link \( a \)
- \( t_a(x_a, y_a) \): Travel cost on link \( a \) as a function of flow and capacity expansion
- \( g_a(y_a) \): Improvement cost function for link ‘a’
- \( B \): Budget (nonnegative real value)
Formulation

**Lower Level problem (LLP)**

Minimize \( TT = \sum_{a \in A} \int_{0}^{x_a} t_a(x_a, y_a) \, dx \)

Subject to:

\[
q_{ij} = \sum_{k \in k_{ij}} f_k^{ij} \quad \forall (i, j) \in IJ
\]

\[
x_a = \sum_{(i,j) \in IJ} \sum_{k \in k_{ij}} \delta_{ak} f_k^{ij}, \quad \forall a \in A
\]

\[
f_k^{ij} \geq 0, \quad \forall k \in k_{ij}, \quad \forall (i, j) \in IJ,
\]

\[
q_{ij} \geq 0, \quad \forall (i, j) \in IJ
\]
### Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$C_a$</td>
<td>The capacity for link $a$</td>
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<tr>
<td>$f_{ij}^r$</td>
<td>Flow on path $r$, connecting each Origin-Destination (O-D) pair $(i,j)$</td>
</tr>
<tr>
<td>$q_{ij}$</td>
<td>Demand between each Origin-Destination (O-D) pair $(i,j)$</td>
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<tr>
<td>$t_a$</td>
<td>Travel time for link $a$</td>
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<tr>
<td>$t_a(x_a,y_a)$</td>
<td>Travel cost on link $a$ as a function of flow and capacity expansion</td>
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<tr>
<td>$x_a$</td>
<td>Flow for link $a$</td>
</tr>
<tr>
<td>$\alpha_a$</td>
<td>Constant, varying by facility type (BPR function)</td>
</tr>
<tr>
<td>$\beta_a$</td>
<td>Constant, varying by facility type (BPR function)</td>
</tr>
<tr>
<td>$\delta_{a,ij}^r$</td>
<td>Binary variable 0,1 {1, if link $a \in A$ is on path $k \in k^{ij}$:0, otherwise}</td>
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<td>Free flow time on link $a$</td>
</tr>
<tr>
<td>$y_a$</td>
<td>Capacity expansion for link ‘a’ (nonnegative real value)</td>
</tr>
</tbody>
</table>
Kth best algorithm for bi-level optimization

- Iteratively solves both upper and lower level problems
- Both upper and lower level can be solved using exact algorithms
- Often provides better solution than evolutionary algorithms (Karoonsoontawong & Waller 2006)

Application:
- Static traffic assignment
- Dynamic traffic assignment
- Other bi-level problem domains
Solution Approach

Initial Traffic Assignment → Base traffic flow

Network planner's problem
Minimize $TSTT = \sum_{a \in A} (x_a t_a(x_a, y_a))$

Pswarm algorithm

User equilibrium problem
Minimize $TT = \sum_{a \in A} \int_0^{x_a} t_a(x_a, y_a) dx$

Slope-based Path Shift-propensity Algorithm (SPSA)

Did users stop responding to improvements?

Yes → Stop

No → Design constraints

Definitional constraint
Demand conservation constraint
Non-negativity constraint

Stop
Slope-based Path Shift-propensity Algorithm (SPSA)

- Algorithm Characteristics:
  - Combines merits of simultaneous and sequential approach
    - Updates path sets for all O-D pair simultaneously
      - partially tackles the problem of order bias
    - Equilibrates one O-D pair at a time
      - leads to faster convergence
  - Incorporates behavioral realism in the flow update process
  - Convergence is theoretically proven
  - Simplicity of execution for easy deployment in practice
- Developed by Kumar and Peeta (2014)
Particle swarm (pswarm) algorithm

- Simulate behavior of particles that attempt to optimally explore some given solution space
- Population of particles is called swarm
- A particle flies in the solution space in search of optimal position
- Particle adjust its position and velocity using its own as well as other particles’ experiences in the population
  - Combines the local search (own experience) with global search (population experience)
- Known to perform better than other global optimization methods such as genetic algorithm
\[ t_a(x_a, y_a) = A_a + B_a \left( \frac{x_a}{C_a + y_a} \right)^4 \]

\[ TSTT(y) = \sum_a (t_a(x_a, y_a) \cdot x_a + 1.5d_a y_a^2) \]

<table>
<thead>
<tr>
<th>Arc \ a</th>
<th>A_a</th>
<th>B_a</th>
<th>C_a</th>
<th>d_a</th>
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### Comparison of Results

<table>
<thead>
<tr>
<th>Case</th>
<th>MINOS</th>
<th>Hooke-Jeeves (H-J)</th>
<th>EDO</th>
<th>GA</th>
<th>Current Study</th>
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<td>Demand =300</td>
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</table>

### Names of heuristics and Sources

<table>
<thead>
<tr>
<th>Names of heuristics</th>
<th>Sources</th>
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</thead>
<tbody>
<tr>
<td>GA</td>
<td>Genetic Algorithm</td>
</tr>
<tr>
<td>H-J</td>
<td>Hooke-Jeeves algorithm</td>
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<tr>
<td>EDO</td>
<td>Equilibrium Decomposed Optimization (Bolzano search)</td>
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<tr>
<td>MINOS</td>
<td>Modular In-core Non linear System</td>
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</table>
Test Network 2

candidate links are marked by red arrow
LL and UL objective functions
Dissimilarity of link flow vectors
## Comparison of Results for Sioux Falls Network

<table>
<thead>
<tr>
<th>Case</th>
<th>H-J</th>
<th>EDO</th>
<th>SA</th>
<th>SAB</th>
<th>GP</th>
<th>CG</th>
<th>QNew</th>
<th>PT</th>
<th>GA</th>
<th>Current Study</th>
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</table>

- **GP**: Gradient Projection method
- **CG**: Conjugate Gradient projection method
- **QNEW**: Quasi-NEWton projection method
- **PT**: PARTAN version of gradient projection method
# Comparison of Results

## Comparison of Results for Sioux Falls Network for different demand level

<table>
<thead>
<tr>
<th>Demand Scenario</th>
<th>SAB</th>
<th>GP</th>
<th>CG</th>
<th>QNew</th>
<th>PT</th>
<th>EDO</th>
<th>IOA</th>
<th>GA</th>
<th>Current Study</th>
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<td>48.78</td>
<td>48.84</td>
<td>48.81</td>
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<td>53.58</td>
<td>48.92</td>
<td>48.15</td>
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<td>1 Itr.</td>
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<td>82.71</td>
<td>82.53</td>
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| GA   | Genetic Algorithm |
| GP   | Gradient Projection method |
| CG   | Conjugate Gradient projection method |
| QNEW | Quasi-NEWton projection method |
| PT   | PARTAN version of gradient projection method |
### Application

<table>
<thead>
<tr>
<th>Network</th>
<th>Nodes</th>
<th>Links</th>
<th>Zones</th>
<th>O-D pairs with non-zero demand</th>
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<tr>
<td>Anaheim</td>
<td>416</td>
<td>914</td>
<td>38</td>
<td>1,416</td>
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<td>Chicago Sketch</td>
<td>933</td>
<td>2,950</td>
<td>387</td>
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<tr>
<td>Washington DC</td>
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<td>144</td>
<td>20,736</td>
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Anaheim

SR-91
I-5
SR-57
SR-55
SR-22
LL and UL objective functions
Dissimilarity of link flow vectors
Anaheim Investment Scenarios

- **Objective Function**
  - Budget (million) vs. Objective Function
  - Budget increases, Objective Function decreases

- **Average Speed**
  - Budget (million) vs. Average Speed
  - Budget increases, Average Speed increases

- **CLM**
  - Budget (million) vs. CLM
  - Budget increases, CLM decreases

- **Average Travel Time**
  - Budget (million) vs. Average Travel Time
  - Budget increases, Average Travel Time decreases
Chicago Sketch
LL and UL objective functions

- LL objective function
  - Minimum value: $1.916 	imes 10^7$
  - Maximum value: $1.922 	imes 10^7$

- UL objective function
  - Minimum value: $1.745 	imes 10^7$
  - Maximum value: $1.765 	imes 10^7$
Dissimilarity of link flow vectors
Chicago Sketch Investment Scenarios

- Objective Function
- Average Speed (mph)
- CLM
- Average Travel Time (min)
Chicago Investment Scenarios

Chicago Sketch Network

Budget = $300 million

Budget = $375 million

Budget = $450 million

Budget = $525 million

Budget = $600 million

Capacity

0.0 - 2500.0
2500.1 - 5000.0
5000.1 - 7500.0
7500.1 - 10000.0
10000.1 - 12500.0

Miles
Montgomery County and Atlanta
Montgomery County Investment Scenarios

UL Objective Function

Average Speed

CLM

Average Travel Time
Atlanta Investment Scenarios

UL Objective Function

Average Speed

CLM

Average Travel Time

Budget (Million)

UL Objective Function (LOG)

Average Speed

Budget (Million)

CLM

Average Travel Time

Budget (Million)
Summary and Conclusion

- In the investment decision making problem
  - One player (one upper level objective function)
  - One year analysis period
  - Kth best algorithm
  - Iterative approach between UL and LL
  - Compared to results from literature
  - Applied the procedure in real life networks
Limitations and Future Research

- Limitations
  - Kth best algorithm may not guarantee optimality
  - Computational time

- Future research
  - Converting single level
  - Considerations of multi year, multi objectives
  - Developing economic performance measures
Q & A

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