Using Error Distributions with Model Output to Acknowledge Prediction Uncertainty: Results using Travel Demand and Transit Flow Models

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Uncertainty in Transportation Demand and Flow Models

Postulates
• There is uncertainty in predictions/forecasts: “Models are off”
• It is better to recognize than ignore the uncertainty

Practice
• Transportation demand/flow models generally produce point estimates

Propose and validate an approach to “add” uncertainty to model point estimates
Recognizing Differences between Models and the “Truth”

• Empirical-based studies
  • Measures of difference (e.g., RMSE) to compare models
  • *Do not provide measure for prediction uncertainty*

• Theoretical/Numerical (Monte Carlo)–based studies
  • Provide measures for prediction uncertainty based on distributions of inputs or parameters
  • *Do not account for model/assumption uncertainty*

• This approach
  • *Use differences between past model-based and observed values to determine distribution of true value, conditional on model output*
Developing Uncertainty Distributions
Difference between Model Values and Observations: $\Delta$

MORPC Network

1990 Link $i$

<table>
<thead>
<tr>
<th>Volume</th>
<th>MORPC Model Output ($M_i$)</th>
<th>Observation ($T_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>26,844 *</td>
<td>29,340 *</td>
</tr>
</tbody>
</table>

$\Delta_i = M_i - T_i$

-2,496

* Ferdous et al. (2011) Comparison of Four Step Versus Tour-Based Models in Predicting Travel Behavior Before and After Transportation System Changes
Determining Difference ($\Delta$), Bias (b), and Error ($\epsilon$) Distributions

$\Delta_1 = -667$

$\Delta_j = 1,707$

$\Delta_i = -2,496$

$\Delta_k = -3,244$

1041 Segments in Ferdinand et al.
Determining Difference ($\Delta$), Bias ($b$), and Error ($\varepsilon$) Distributions

Differences Between Models and Observations

$$\Delta = M - T_{\text{obs}}$$

Bias: $$b = -E[\Delta] = -1,155$$

Unbiased Error

$$\varepsilon = \Delta + b$$
Determining Bias Distribution

\[ b^1 = -E[\Delta^1] \]

\[ b^\gamma = -E[\Delta^\gamma] \]
Prediction/Forecast on Link i: $F_i(T_i|M_i)$

Model for 2005

$M_i = 58,071$

$F_b$ and $F_\varepsilon$ from 1990 and 2000

Prediction for 2005

$F[T_i|M_i=58,071]$
Validation Study

• Use subset of model/observation data to estimate bias and error distribution

• Use estimated bias and error distributions with remaining model output to produce uncertainty in model predictions/forecasts

• Use observations for remaining data (“known outcomes of prediction/forecast”) with modeled uncertainty to determine empirical distributions of probabilities of observations

• Compare empirical distributions to theoretical distributions
Prediction/Forecast on Link i: $F_i(T_i|M_i)$

Model for 2005

$M_i = 58,071$

$F_b$ and $F_\epsilon$ from 1990 and 2000

Bias $F_{b(i)}(b)$

Error $F_{\epsilon(i)}(\epsilon)$

Prediction for 2005

$F_{[b]}$ $F_{[\epsilon]}$ $F_{[T_i|M_i=58,071]}$ $T_i[\text{Veh}]$
Validation Logic: Probabilistic Forecast of Observation $T_{obs}$ on Link $i$

In 2005
Model link $i$: $M_i = 58,071$
Observation link $i$: $T_{obs}^i = 62,025$

$F_i(62,025) = 0.60$

$T_{obs}^i = 62,025$
In 2005
Model link $j$: $M_j = 30,998$
Observation link $j$: $T^{obs}_j = 35,860$

$F_j(35,860) = 0.62$

$T^{obs}_j = 35,860$
In 2005
Model link k: $M_k = 59,050$
Observation link k: $T_{\text{obs}}^k = 54,025$
Monte Carlo Logic:
*Well-calibrated uncertainty should produce points around 45° line*

Metrics of discrepancy w/ 45° line
- AAD: |Avg. Dif. (45°, pts.)|
- MD: |Max. Dif. (45°, pts.)|
- Area (45°, pts.)

*Larger metric values imply poorer empirical distributions*
Empirical Applications

• Link volumes from traffic assignment
  • Mid-Ohio Regional Planning Commission model outputs and observations
  • Values from Ferdous et al. (2011): Tour-based model

• Bus passenger OD (B2A) flows from estimations based on boarding and alighting data
  • The Ohio State University Campus Transit Lab OD flow observations (http://transitlab.osu.edu/campus-transit-lab)
  • Boarding and alighting data from observations used with Iterative Proportional Fitting (IPF) method to produce model estimates
Traffic Assignment Validation
Using Ferdous et al. MORPC Data

- Model/Observation years: 1990, 2000, 2005
- Calibrate using two years to predict third year: All (3) combinations
- Calibrate one bias and one error distribution using all segments: “Aggregated Calibration”
- Pool results

![Graph showing Aggregated Calibration with statistics: AAD: 0.058, MD: 0.130, Area: 0.057]
Traffic Assignment Validation
Aggregated Calibration for Segmented Predictions

Freeway

Cumulative of $F(T^{\text{obs}}_i)$

AAD: 0.087
MD: 0.208
Area: 0.086

Major Road

Cumulative of $F(T^{\text{obs}}_i)$

AAD: 0.031
MD: 0.061
Area: 0.030

Minor Road

Cumulative of $F(T^{\text{obs}}_i)$

AAD: 0.088
MD: 0.178
Area: 0.090

Local Road

Cumulative of $F(T^{\text{obs}}_i)$

AAD: 0.123
MD: 0.235
Area: 0.133
Traffic Assignment Validation
Aggregated vs. Segmented (Bias and Error) Calibration

Aggregated Calibration

- AAD: 0.058
- MD: 0.130
- Area: 0.057

Segmented Calibrations by Functional Class

- AAD: 0.034
- MD: 0.091
- Area: 0.034

Calibrating Bias and Error Distributions for Each Functional Class Improves Results
Empirical Applications

• Link volumes from traffic assignment
  • Mid-Ohio Regional Planning Commission model outputs and observations
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• Bus passenger OD (B2A) flows from estimations based on boarding and alighting data
  • The Ohio State University Campus Transit Lab OD flow observations (http://transitlab.osu.edu/campus-transit-lab)
  • Boarding and alighting observations for six academic terms
  • Model output using Iterative Proportional Fitting (IPF) method
Calibrating Bias and Error Distributions for High/Low Volume Cells Improves Results
Conclusions

• Preliminary validation studies indicate the approach is capturing uncertainty appropriately
• Additional studies needed to refine approach and produce more robust validation studies (“spin-off” research investigations also envisioned)
• Request for agency model validation data
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