# Assigning User Class Link and Route Flows Uniquely to Urban Road Networks[1]

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Abstract

Static user-equilibrium traffic assignment, the standard method for predicting traffic flows on urban road networks, is based on the principle that drivers choose their own least cost routes from their origins to their destinations. This principle may be formally described by user-equilibrium conditions, which correspond to the optimal solution to a nonlinear minimization problem. Although the total flows on links of the road network are uniquely determined by this formulation, multiple user-class link flows, as well as route flows, are not. Nevertheless, professionals frequently use class link flows output by their software for project evaluation, and class route flows to perform select link analyses. These applications may result in misleading findings, for example, in scenario analyses. An additional assumption may be imposed to determine these user class link flows and route flows uniquely. A possible assumption is the condition of proportionality, namely that the proportion of vehicles on each of two alternative, equal-cost segments (sequences of links) should be the same regardless of their user class, origin or destination.

This paper compares two assignments in which the order of the car and truck trip matrices was specified as car followed by truck, and truck followed by car. Although the total link flows for the two assignments are effectively equal, substantial differences exist between the user class link flows determined by the different orderings of the matrices. Post-processing to impose the condition of proportionality on the class O-D flows removes about 90% of these differences. Analyses of class link flows of cars and trucks before and after imposing the condition of proportionality for one of these assignments reveal that about half of all links experience differences in user class flows, ranging up to +/–300 vph. These findings offer insights into the magnitude of the differences arising from the non-uniqueness of class link flows, and identify which links are subject to differences. Imposing the condition of proportionality in multi-class traffic assignments is recommended in travel forecasting practice.

Keywords: static user-equilibrium traffic assignment; multi-class link flows; condition of proportionality; order of assigned trip matrices

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## 1. Introduction

Capabilities to solve the user-equilibrium traffic assignment problem on large road networks for transportation planning practice have advanced substantially during the past decade. New solution algorithms, and related improvements in commercial software systems, have increased the precision of assignments achievable by professional practitioners. Moreover, network size and detail and the number of classes assigned continue to expand in response to the needs of practice, as enabled by ongoing increases in the size and speed of computers.

One implication of these advances is that professional practitioners are seeking to understand the attributes of their assignments in more depth. One tool for exploring an assignment is select link analysis. Such analyses seek to identify the origins and destinations of route flows through a selected link, and to compare these findings across scenarios. Other analyses compare class link flows in a base network with a proposed improvement. For example, the effectiveness of a road bypass on the routing of truck flows might be examined. Our discussions with practitioners suggest that such tools are intensively utilized in everyday planning practice.

Many practitioners are aware that they face a dilemma when utilizing such tools, namely that route flows and class link flows may not be uniquely determined by the solution to the standard user-equilibrium assignment problem. This non-uniqueness is a property of the mathematical formulation of the assignment problem itself, regardless of the solution algorithm. Intuitively, the reason is that link travel times are functions of total link flows, and not functions of class or route flows. Therefore, vehicle flows can be swapped among user classes or routes, leaving the total link flows unchanged. Faced with the alternative of using the computed route and class link flows in an analysis, or doing nothing, many analysts are inclined to proceed cautiously, being cognizant of possible discrepancies or inconsistencies in their results.

If one desires to confront the issue of uniqueness of class link flows and route flows, however, that objective may be achieved by imposing an additional assumption on the assignment. A possible assumption is the condition of proportionality, namely that the proportion of vehicles on each of the two alternative, equal-cost segments (sequences of links) should be the same regardless of their user class, origin or destination.

The condition of proportionality can be applied with a post-processing procedure to solutions determined by an algorithm defined on a bush-based representation of a road network; examples are the OUE algorithm in TransCAD (Slavin et al., 2011; Caliper, 2013) and the LUCE algorithm in Visum (Gentile, 2012; PTV, 2013). The condition of proportionality also enabled Bar-Gera (2010) to design a new algorithm, Traffic Assignment by Paired Alternative Segments (TAPAS), to compute route flow and two-class link flow solutions with proportionality directly.

An important research question concerns whether proportionality is observed in reality. In a preliminary analysis, route flows over an expressway network in Guangdong Province, China, appear to exhibit attributes of proportionality. Testing the proportionality hypothesis requires much more extensive data collection and analysis, which is now being initiated by Hu (2013).

Until more is known about proportionality in reality, the imposition of the condition on certain traffic assignments enables the comparison of assignments to proceed with the assurance that computed differences in route and class link flows among scenarios are valid. Without imposing this condition, these differences could be artifacts of the manner in which the assignment was set up, as illustrated in this paper.

Until now, the effects of imposing proportionality by post-processing a traffic assignment have not been studied. (Bar-Gera et al. (2012) presented select link analyses comparing O-D flows with proportionality, as found by TAPAS, with link-based and route-based flows.) The objective of this paper, therefore, is to analyze these effects on class link flows at three levels: 1) differences in class link flows without and with proportionality on a small subnetwork for two related assignments in Section 4; 2) the effects of imposing proportionality on cars link flows for the entire network in Section 5; 3) changes in car O-D flows for a selected arterial link in Section 6. Sections 2 and 3 review the literature and describe the data; Section 7 offers conclusions and recommendations for practitioners.

## 2. Literature Review

The standard method for predicting traffic flows on congested urban road networks, static user-equilibrium (UE) traffic assignment, is based on an assumption that drivers seek their own least cost routes from their origins to destinations. This concept corresponds to a UE state in which all used routes have equal costs and no unused route has a lower cost for every origin-destination pair (Wardrop, 1952). This problem can be formulated as a convex optimization problem with linear constraints (Beckmann et al., 1956). A property of Beckmann's formulation, and of algorithms devised for its solution, is that route flows may not be uniquely determined. If two or more classes of vehicles are assigned (e.g., cars and trucks) with the same generalized cost (impedance) function, the class link flows also may not be uniquely determined.

Solution methods for the standard traffic assignment problem, in which link travel times/costs depend only on their own flows, have advanced steadily since the late 1960s. Earlier methods were based on the intuitive notion that drivers seek the shortest route to their destinations. Until the mid-1970s, however, many practitioners were unaware of Beckmann's UE formulation and its implications for designing algorithms that converge towards the UE state. Examples of early methods are the non-convergent, iterative capacity-restraint method of the Bureau of Public Roads (US DOC, 1964) and the incremental method devised for TRANPLAN (CDC, 1965), which does converge towards the UE state, albeit very slowly. See Patriksson (1994) and Ortúzar and Willumsen (2011) for further details about the static traffic assignment problem, and Boyce (2007) concerning its history from 1952 onwards.

Convergent assignment methods based on Beckmann's formulation began to emerge in the late 1960s and early 1970s in Ph.D. thesis research in operations research and civil engineering (Bruynooghe et al., 1969; Dafermos and Sparrow, 1969; Nguyen, 1974; LeBlanc et al., 1975; Evans, 1976). The first convergent method to be widely applied was based on the linearization method of Frank and Wolfe (1956). One of the originators of this assignment algorithm was LeBlanc. Concurrently, Evans (1976) offered a generalization of the same formulation to include trip distribution, and rigorously analyzed the mathematical properties of a partial linearization algorithm. All of these algorithms solved the assignment problem in terms of link flows, and are called link-based. Link-based methods rather quickly replaced the older heuristic methods during the 1980s, when software systems for mini- and micro-computers began to be offered.

Today, the leading commercial travel forecasting software systems (Cube, Emme, Saturn, TransCAD and Visum) include various link-based solution methods. In addition, a convergent route-based method (Bothner and Lutter, 1982) is offered in Visum. A drawback of link-based methods is the computational effort required to solve the assignment problem precisely may be excessively large. Even so, recent advances in these methods continue to be incorporated into some software systems (Florian et al., 2009; Mitradjieva and Lindberg, 2013).

Refinements to these solution methods continued through the 1990s. When Bar-Gera (2002) proposed his origin-based assignment algorithm, the situation began to improve. His method offered several advances; one was the organization of the search on an acyclic, origin-based subnetwork defined for each origin zone, earlier called a bush by Dial (1971). Bar-Gera's algorithm was able to achieve precise solutions of the assignment problem for the first time, although the computation times were relatively long. Later, Dial (2006) proposed his bush-based algorithm, which solved the assignment problem precisely and more quickly.

The precision of the solution needed for practice depends on its use. One of the main applications of travel forecasts is to compare scenarios. Such comparisons are only valid if the precision of each solution is substantially better than the differences among the scenarios. Boyce et al. (2004) explored the effects of solution precision on scenario differences, recommending that the relative gap, a standard measure of convergence, should be less than 1E-4.

None of these advances directly addressed the question of the non-uniqueness of route flows, or class link flows in a multi-class assignment. Bar-Gera (2006) returned to this question, which he had posed in his 1999 Ph.D. thesis, exploring the issue theoretically and computationally. In 2008, he proposed a new algorithm, Traffic Assignment by Paired Alternative Segments (TAPAS) based directly on the condition of proportionality (Bar-Gera, 2010). Shortly thereafter, two bush-based algorithms with post-processing for proportionality were offered: Origin User Equilibrium (OUE) was introduced into TransCAD, and Local User Cost Equilibrium (LUCE) was added to Visum. These two algorithms with post-processing, plus TAPAS, represent the current state of the art for solving UE traffic assignment route and class link flows uniquely.

The non-uniqueness of class link flows under the UE assumption may be illustrated with a simple example. Suppose that the total vehicle link flows per hour shown in Fig. 1 represent a perfect UE solution with identical travel times over a pair of alternative segments: [2, 3, 5] and [2, 4, 5]. How many cars and trucks (in car-equivalent units) will use each segment? Three solutions for the vehicle flows on each of the four class and route combinations in this network are shown in Table 1, each solution corresponding exactly to the total link flows.

If one wants to determine uniquely how many cars and trucks use each link, an additional assumption is required. A plausible assumption is the condition of proportionality, namely that the proportion of cars and trucks on each of the two alternative segments should be the same. This condition is observed in class link flow solution h* in Table 1, as a ratio of 1 to 3 is found for the car segments (25 to 75) as well as for the truck segments (15 to 45), corresponding to the total link flows (40 to 120).

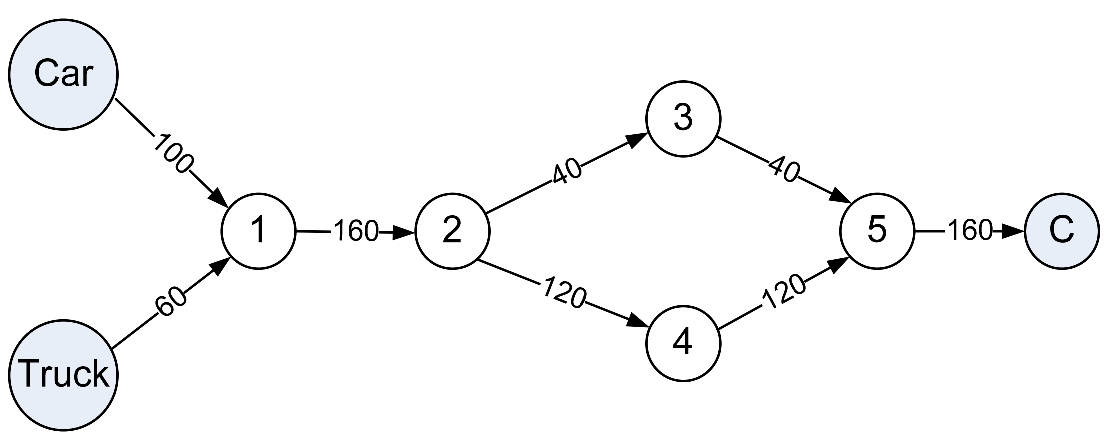


**Fig. 1.** Link flows on a pair of alternative segments.

**Table 1**
Alternative class link flow interpretations of total link flows in Fig. 1.

| Node pair | Class-segment | h* | $h_1$ | $h_2$ |
|---|---|---|---|---|
| Car-C | Car-1-2-3-5-C | 25 | 40 | 0 |
| Car-C | Car-1-2-4-5-C | 75 | 60 | 100 |
| Truck-C | Truck-1-2-3-5-C | 15 | 0 | 40 |
| Truck-C | Truck-1-2-4-5-C | 45 | 60 | 20 |

Source: Based on Boyce et al. (2010, p. 7)

Five reasons to adopt the condition of proportionality are: 1) it is a reasonable condition that is easily understood; 2) it offers consistent treatment of vehicles in a forecast, which is important for equity; 3) it results in stable solutions with respect to model inputs; 4) satisfaction of the condition of proportionality can be tested computationally for any solution; and 5) given suitable data, tests of the proportionality hypothesis appear to be possible. These properties contribute to a solution's usefulness, especially as the only alternative is to choose arbitrarily one of the many alternative UE solutions. The remainder of the paper seeks to assess the characteristics of class link flows through examples and charts illustrating how user class link flows differ under various conditions.

## 3. Trip Matrices and Representation of the Road Network

Results presented below are based on the assignment of car and truck matrices representing the morning peak period (6:30 - 8:30 am) of the Chicago region in 1990. The Chicago regional zone system for 1990 consisted of 1790 zones (Fig. 2a); the road network had 12,982 nodes and 39,018 links (Fig. 2b). Total truck trips were estimated by the Chicago Area Transportation Study (CATS) to be 0.45 million passenger-car equivalents per hour (pceph). Total regional person trips by car and transit were estimated by Bar-Gera and Boyce (2007) to be 1.51 million persons per hour, or 0.98 million vehicles per hour, using an origin-destination-mode choice function, given the truck trip matrix. Exogenous estimates of the total weekday person flows departing from and arriving at each zone during the two-hour peak period were based on estimates of weekday flows prepared by CATS. The origin-destination person flows by mode were determined with a negative exponential (logit) function, $d_{pq}^{m} = R_p S_q \exp\left(-\beta c_{pq}^{m}\right)$, where $d_{pq}^{m}$ is the origin-destination-mode flow in persons per hour from origin $p$ to destination $q$ by mode $m$ (car, transit); $R_p, S_q$ are balancing factors ensuring that the origin and destination constraints on total departing and arriving flows are met; $c_{pq}^{m}$ is the origin-destination generalized cost by mode $m$ denominated in minutes; and $\beta$ is a cost sensitivity parameter. Use of this function means that origin-destination flows are positive and real-valued, with many flows being less than one person per hour. The generalized travel costs on which the car flows were based are the endogenous UE times and travel distances consistent with the model-determined car flows, the exogenous truck trip matrix, and fixed travel times and fares for transit.

A key parameter of the logit function is the sensitivity of travelers to generalized costs. A larger value of the parameter $\beta$ means travelers are highly sensitive to travel times, distances and fares, whereas a smaller value means they are less sensitive. For smaller values, trips lengthen in time and distance, which increases congestion over the road network, and shifts travelers to transit, as compared with larger values. In the results presented here, a relatively large value

(0.20) was used. The mean travel time for this value for interzonal car travel is about 20 minutes, with a corresponding mean interzonal transit travel time of about 27 minutes.

In the City of Chicago, two expressways have car-only express lanes. In addition, trucks cannot use the Lake Shore Drive, a 24 km multi-lane, grade-separated roadway along the shore of Lake Michigan. Trucks are also restricted from a boulevard system in the City of Chicago and from certain arterial roads in suburban neighborhoods. Altogether, trucks are restricted from 562 links of the road network. Related details and analyses may be found in Boyce (2012). Given these two trip tables for cars and trucks, assignments were solved with Visum for user class generalized link costs defined as travel time only.

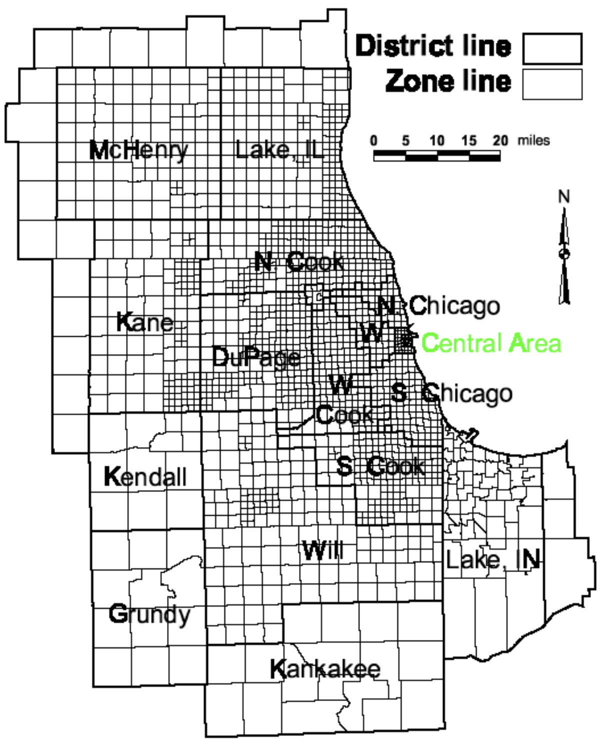


Fig. 2a. Chicago regional zone system.
Source: Chicago Area Transportation Study

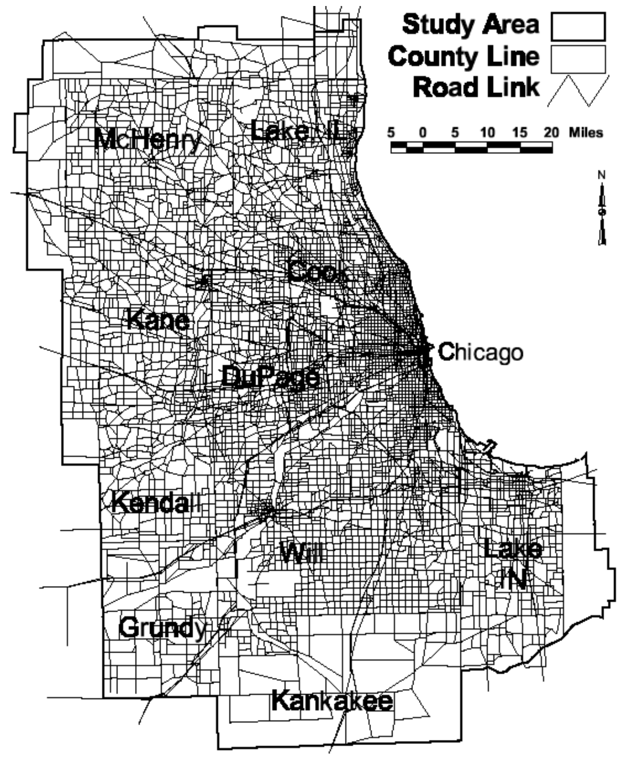


Fig. 2b. Chicago regional road network.

## 4. Class Flows over a Pair of Alternative Segments

To explore the effects of imposing the condition of proportionality on car and truck flows over a network, we consider a typical pair of alternative segments located near North Avenue in Chicago, two miles north of the Central Area, as shown in Fig. 3. This analysis illustrates how the class link flows can be rather arbitrary if proportionality is not imposed. The example was originally motivated by the construction of a new bridge on North Avenue connecting nodes 6380 and 6389. Suppose that an analyst wished to examine the truck flows using this bridge in a two-class assignment of cars and trucks. In the UE solution, the North Avenue bridge is a link in Segment 1.

Two assignments were performed with the LUCE option in Visum. In the initial solution of the first assignment, the car matrix was assigned first, followed by the truck matrix (designated as car/truck); then the solution was iterated to a relative gap of less than 1E-8. In the second assignment, the order of the assigned matrices was reversed (designated as truck/car). The class link flow arrays were saved from each assignment. Then, the class link flow arrays from each assignment were adjusted by imposing the condition of proportionality. Imposing proportionality on the solution increased the solution time by only 10% for these trip matrices and network.

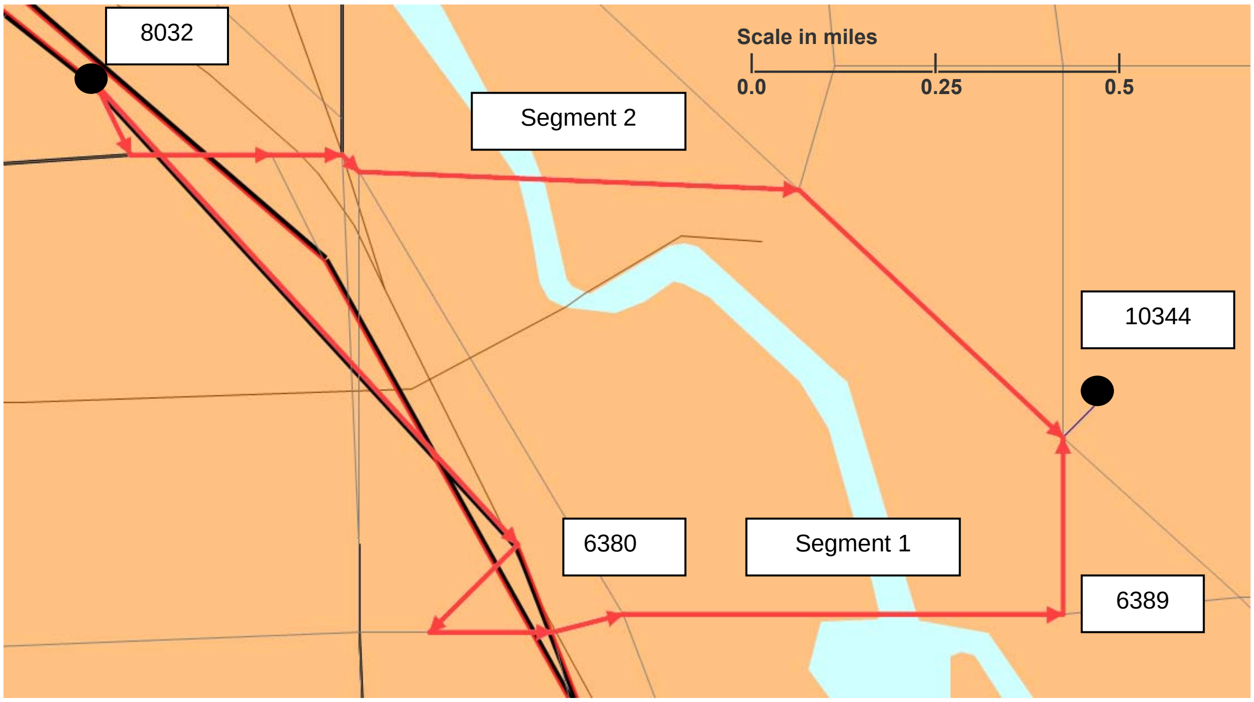


**Fig. 3.** Segments 1 and 2 connecting node 8032 to node 10344.

The total class flows over the two segments are shown in Table 2 for the two assignments. The top half of Table 2 shows the total segment flows when the order of the assignment is car/truck. The bottom half shows the total segment flows when the order of the assignment is truck/car. The left side of the table shows the total flows without proportionality. The right side shows the total flows with proportionality, demonstrating that the two assignments with proportionality have the same solution.

The upper left side shows the flows over the two segments without proportionality for the car/truck ordering: 93% of car flows used Segment 1, compared to only 26% of truck flows. The lower left side with the order of the trip matrices reversed shows only 55% of car flows used Segment 1, compared to 92% of truck flows. Hence the segment flows by class are dramatically different for the two assignments. After proportionality is imposed, as shown on the right side of Table 2, the car and truck flows each have 69% on Segment 1, an increase in the Segment 1 truck flows of 89 vph for the car/truck order, and a decrease of 50 vph for the truck/car order.

**Table 2**
Total segment flows without and with proportionality by order of assignment.

| order of trip matrices in the assignment: car/truck | | | | | | | |
|---|---|---|---|---|---|---|---|
| | flows without proportionality (vph) | | | flows with proportionality (vph) | | | class flow |
| segment | 1 | 2 | 1/total (%) | 1 | 2 | 1/total (%) | (vph) |
| cars | 338 | 25 | 93 | 249 | 114 | 69 | 363 |
| trucks | 56 | 155 | 26 | 145 | 66 | 69 | 211 |
| total | 394 | 180 | 69 | 394 | 180 | 69 | 574 |
| order of trip matrices in the assignment: truck/car | | | | | | | |
| | flows without proportionality (vph) | | | flows with proportionality (vph) | | | class flow |
| segment | 1 | 2 | 1/total (%) | 1 | 2 | 1/total (%) | (vph) |
| cars | 199 | 164 | 55 | 249 | 114 | 69 | 363 |
| trucks | 195 | 16 | 92 | 145 | 66 | 69 | 211 |
| total | 394 | 180 | 69 | 394 | 180 | 69 | 574 |

Flows are rounded to integers.

Table 3 shows the number of O-D pairs that comprise each segment flow by class and order of the assignment. The column, either/both, indicates how many O-D pairs are found on either segment, and how many are found on both segments. The number of O-D pairs is substantially different by order without proportionality, but the same with proportionality. The differences are especially large for Segment 2. The number of O-D pairs with proportionality imposed shows a shift in flows from one segment to both segments: 670 pairs for cars and 256 pairs for trucks.

**Table 3**
Number of O-D pairs using each segment without and with proportionality by order of assignment.

| order of trip matrices in the assignment: car/truck | | | | | | |
|---|---|---|---|---|---|---|
| | O-D pairs without proportionality | | | O-D pairs with proportionality | | |
| segment | 1 | 2 | either/both | 1 | 2 | either/both |
| cars | 668 | 392 | 670/390 | 670 | 670 | 670/670 |
| trucks | 204 | 245 | 256/193 | 256 | 256 | 256/256 |
| total | 678 | 483 | 680/481 | 680 | 680 | 680/680 |
| order of trip matrices in the assignment: truck/car | | | | | | |
| | O-D pairs without proportionality | | | O-D pairs with proportionality | | |
| segment | 1 | 2 | either/both | 1 | 2 | either/both |
| cars | 624 | 641 | 670/595 | 670 | 670 | 670/670 |
| trucks | 256 | 131 | 256/131 | 256 | 256 | 256/256 |
| total | 649 | 655 | 680/624 | 680 | 680 | 680/680 |

To illustrate the class flows at the O-D level, Figs. 4a and 4b show car flows without proportionality and with proportionality. The horizontal axes show the O-D flows on Segment 1, while the vertical axes show the O-D flows on Segment 2. The number of O-D pairs is shown in Table 3. Triangles represent O-D flows without proportionality, and squares represent O-D flows with proportionality. The total segment flows for each solution are shown at the top of each chart. In each chart, the squares form a diagonal line; therefore, the segment flows for all O-D pairs have the same proportions. As shown in Table 2, the slope of the lines formed by the squares is 0.69.

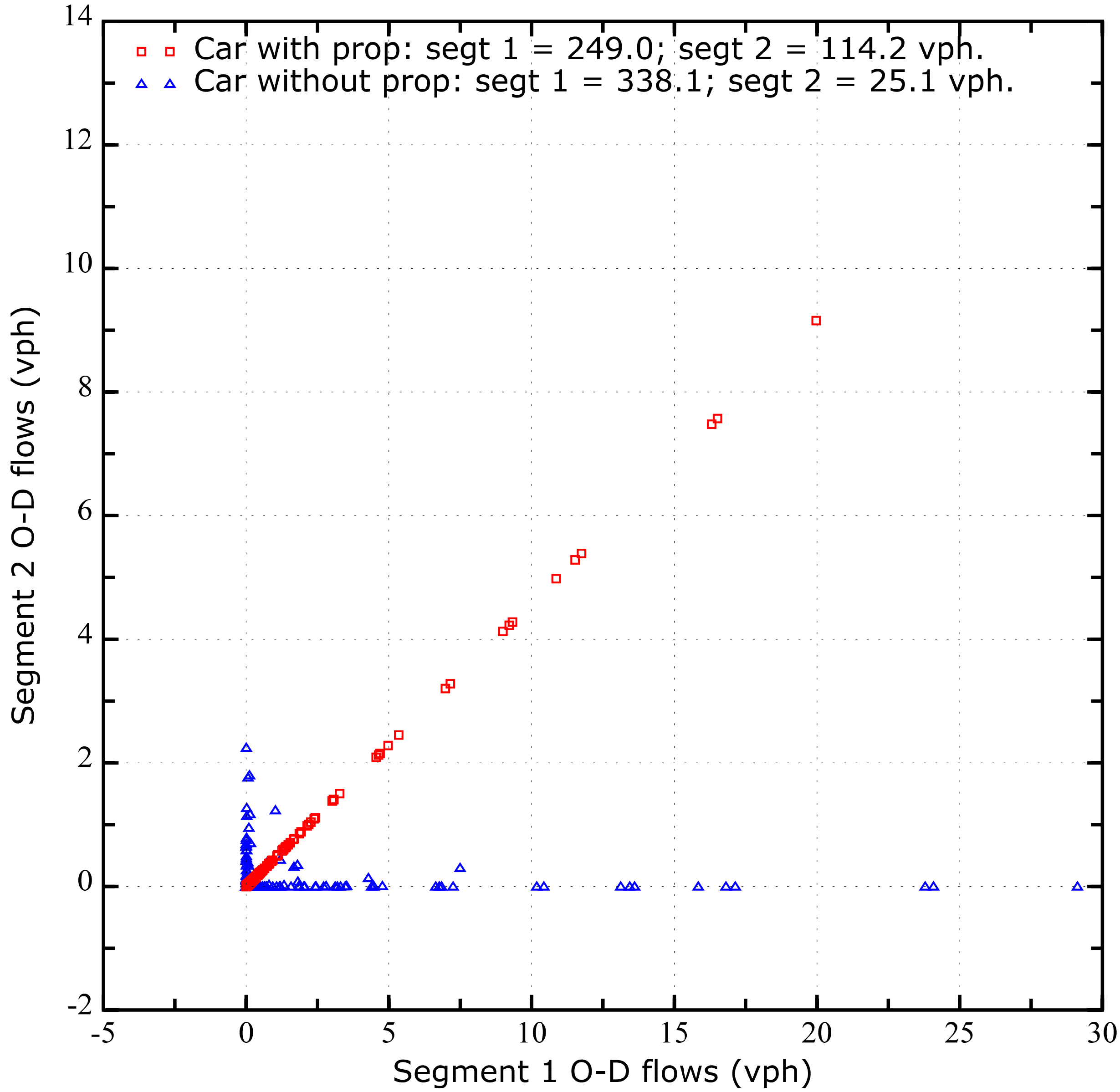


**Fig. 4a.** Car O-D flows by segment: order - car/truck.

In contrast to the squares, most of the triangles lie on one of the two axes. Therefore, these O-D flows are assigned to only one segment. Points lying on the axes may be termed 'all or nothing' with regard to the choice of segment. The same charts (not shown) were drawn for truck flows to confirm that the truck flow patterns are similar.

These two figures are helpful in visualizing the differences in the class flows over a pair of segments, without and with proportionality, for the two orderings of the trip matrices. Since class link flows are sometimes used for plan evaluation and policy analysis, practitioners are advised to consider carefully whether to utilize class link flows for which the condition of proportionality has not been imposed.

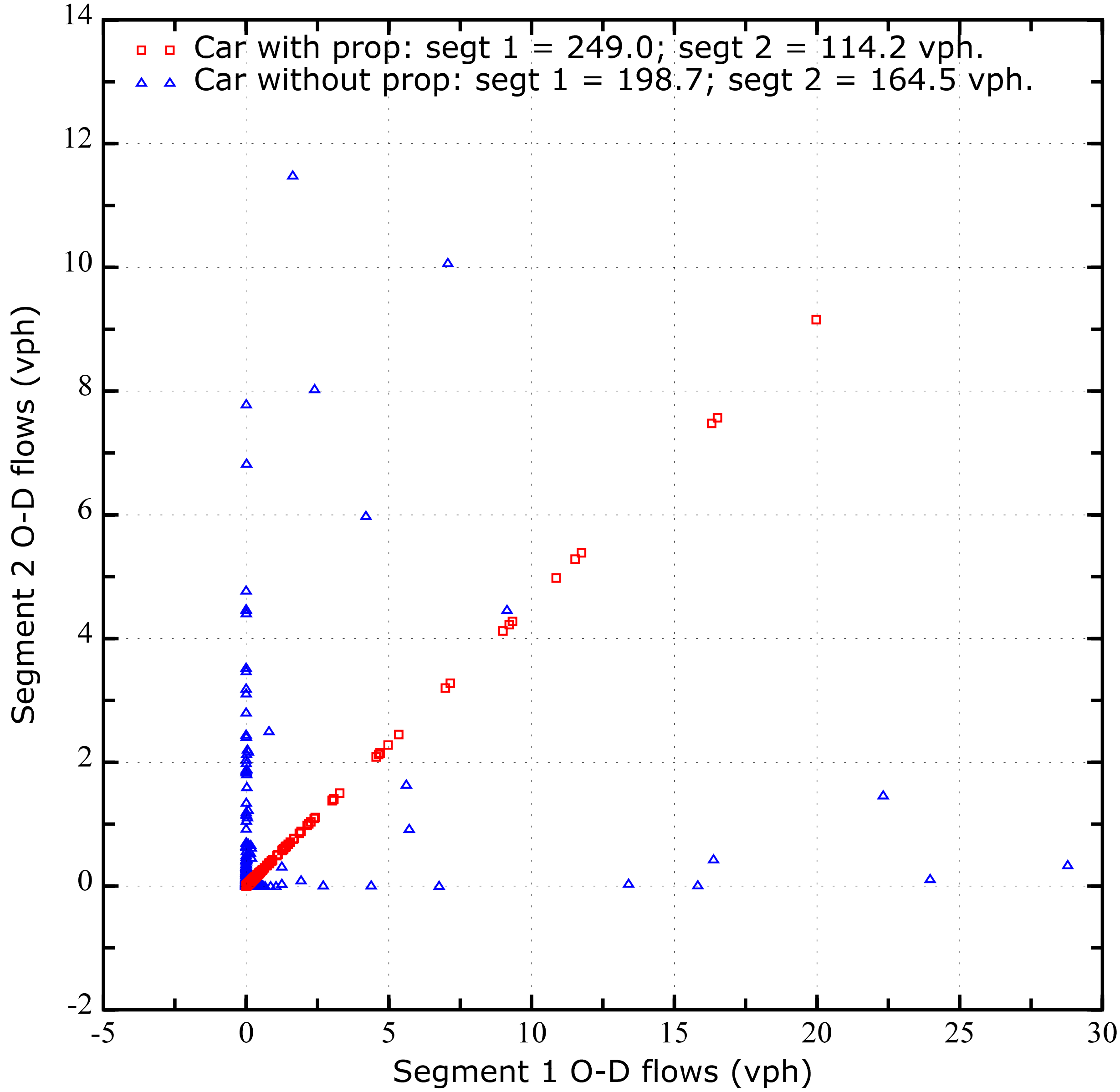


**Fig. 4b.** Car O-D flows by segment: order - truck/car.

## 5. Effects of Proportionality on Class Link Flows in the Network

In this section, differences in class link flows, with and without proportionality, are examined for all links in the network. As an example, first consider the two links shown in Fig. 5, an expressway link and an adjacent arterial link. Then, findings for all car link flows from the assignment with trip matrices in the order of car/truck, are presented. Differences in class link flows with and without proportionality are shown with charts and histograms in Figs. 6-8.

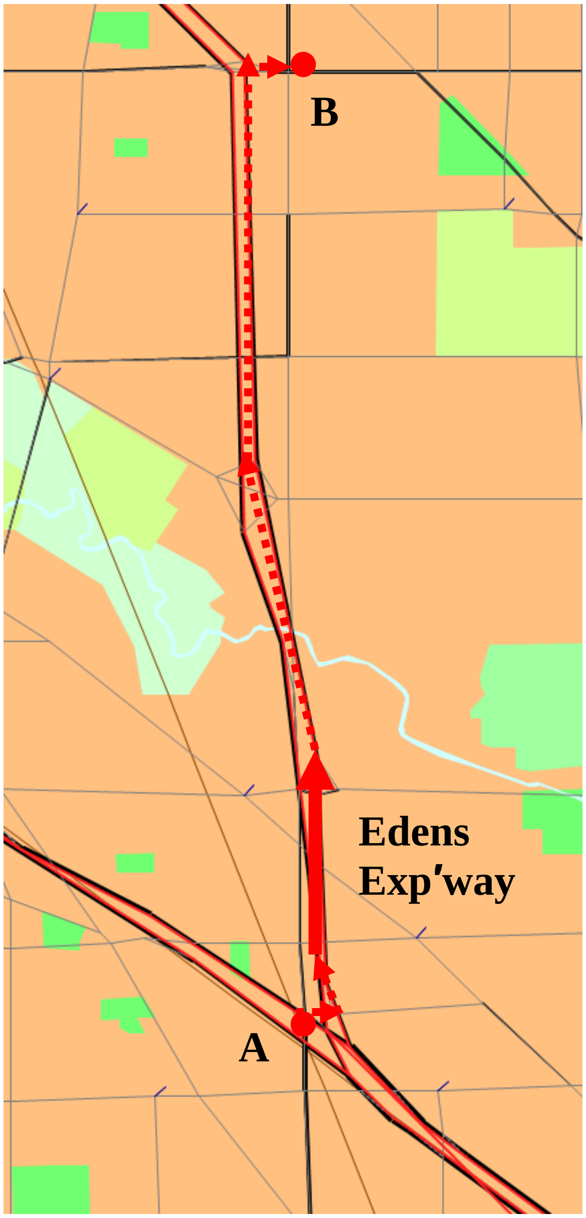


**Fig. 5a.** Edens Expressway northbound link and alternative segment from A to B.

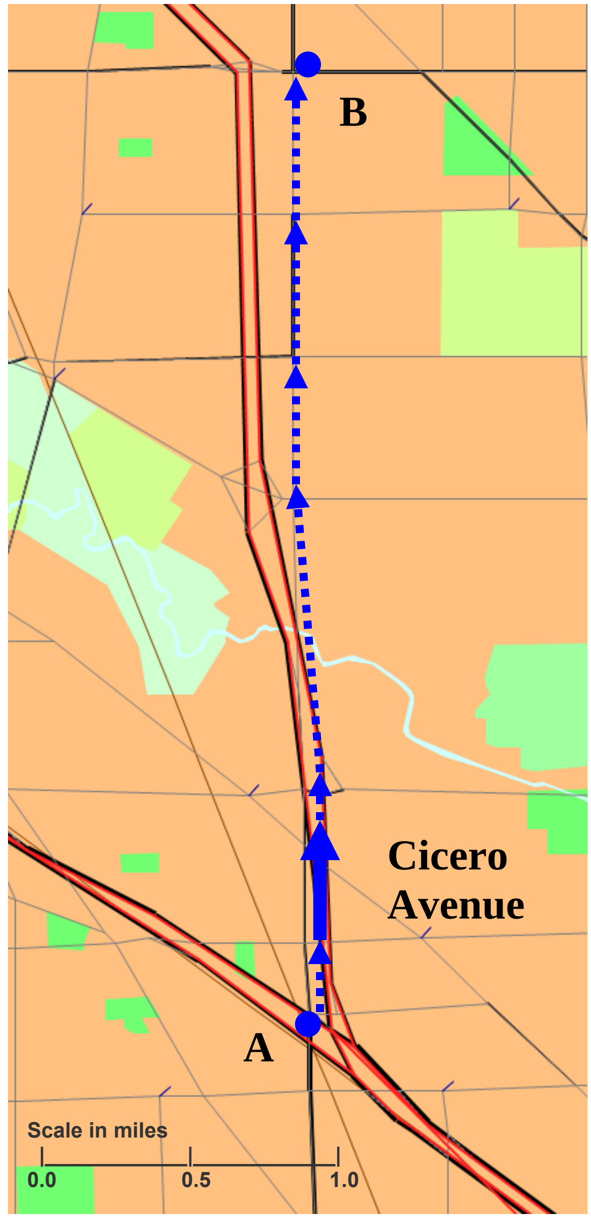


**Fig. 5b.** Cicero Avenue northbound link and alternative segment from A to B.

To illustrate the effect of proportionality on class link flows, consider the two adjacent links (solid arrows) and a related pair of alternative segments (dashed sequences of arrows) in northwest Chicago, shown in Fig. 5. The diverge and merge nodes of the pair of alternative segments are points A and B. The class link flows without and with proportionality for cars and trucks are shown in Table 4. Truck link flows are given in passenger car equivalent (pceph) units.

These flows illustrate the effect of imposing proportionality on class link flows. For the expressway link, the difference (delta) in car and truck flows with and without proportionality is 203 vph, or 12% of the car flow with proportionality. For the arterial link, the difference in car and truck flows with and without proportionality is 199, or 17% of the car flow with proportionality. The total flows on each link are shown in the last column; the link capacities are 5,400 and 2,040 vph, respectively.

These changes in car and truck flows are simply the result of imposing proportionality on the original UE assignment, and consist of thousands of swaps of class and O-D-route flows between 157 pairs of alternative segments that include one or both of these two links, as described in the paragraph preceding Fig. 1. The expressway link is included in 101 segments, the arterial link in 45 segments, and both links in 11 pairs of alternative segments. Altogether, the assignment solution includes 7,266 pairs of segments. At this summary level, this result shown in Table 4 is strictly computational, there being no way to express it analytically.

**Table 4**
Flows on two links without and with proportionality.

| roadway link (solid arrow): | car flow (vph) | | truck flow (pceph) | | delta = \|w – wo\| | delta/ with-car | delta/ with-trk | total flow |
|---|---|---|---|---|---|---|---|---|
| proportionality: | without | with | without | with | (vph) | % | % | (vph) |
| Edens Expressway | 1,942 | 1,739 | 4,747 | 4,950 | 203 | 12 | 4 | 6,689 |
| Cicero Avenue | 946 | 1,145 | 1,292 | 1,093 | 199 | 17 | 18 | 2,238 |

Flows in this table were computed with Visum. Flows are rounded to integers.

As a further extension to this example, the class flows with proportionality over the pair of segments in Fig. 5 are shown in Table 5, together with the segment proportions for class and total flows. This result was found with TAPAS, which reports the class link flows over each pair of alternative segments. This result is not available to the user of Visum.

The properties of the proportionality adjustment are as follows: 1) only links included in a pair of alternative segments can be adjusted for proportionality; 2) the proportionality adjustment modifies the proportion of the route and class flows over each segment, but does not change the total flow of a link; 3) the proportion of class flows comprising a link's total flow is a result of adjustments for all PASs that include that link.

**Table 5**
Flows on a pair of alternative segments with proportionality.

| segment: | car flow (vph) | truck flow (pceph) | proportion by segment (%) | | | segment flow (vph) |
|---|---|---|---|---|---|---|
| | | | car | truck | total | |
| Edens Expressway | 173 | 216 | 86 | 86 | 86 | 389 |
| Cicero Avenue | 28 | 35 | 14 | 14 | 14 | 62 |

Flows in this table were computed with TAPAS (Bar-Gera, 2010). Flows are rounded to integers.

To extend this example to the entire network, car link flow differences, defined as car link flow with proportionality less car link flow without proportionality, are plotted versus the car link flow with proportionality in Fig. 6. As shown in Table 4, the effect of imposing proportionality shifts an equal amount of flow between car and truck flows. Thus, the truck link flow differences are equal to the negative of the car link flow differences, and need not be shown. Fig. 6 shows the effect of imposing proportionality mainly pertains to links with class flows less than 3,000 vph.

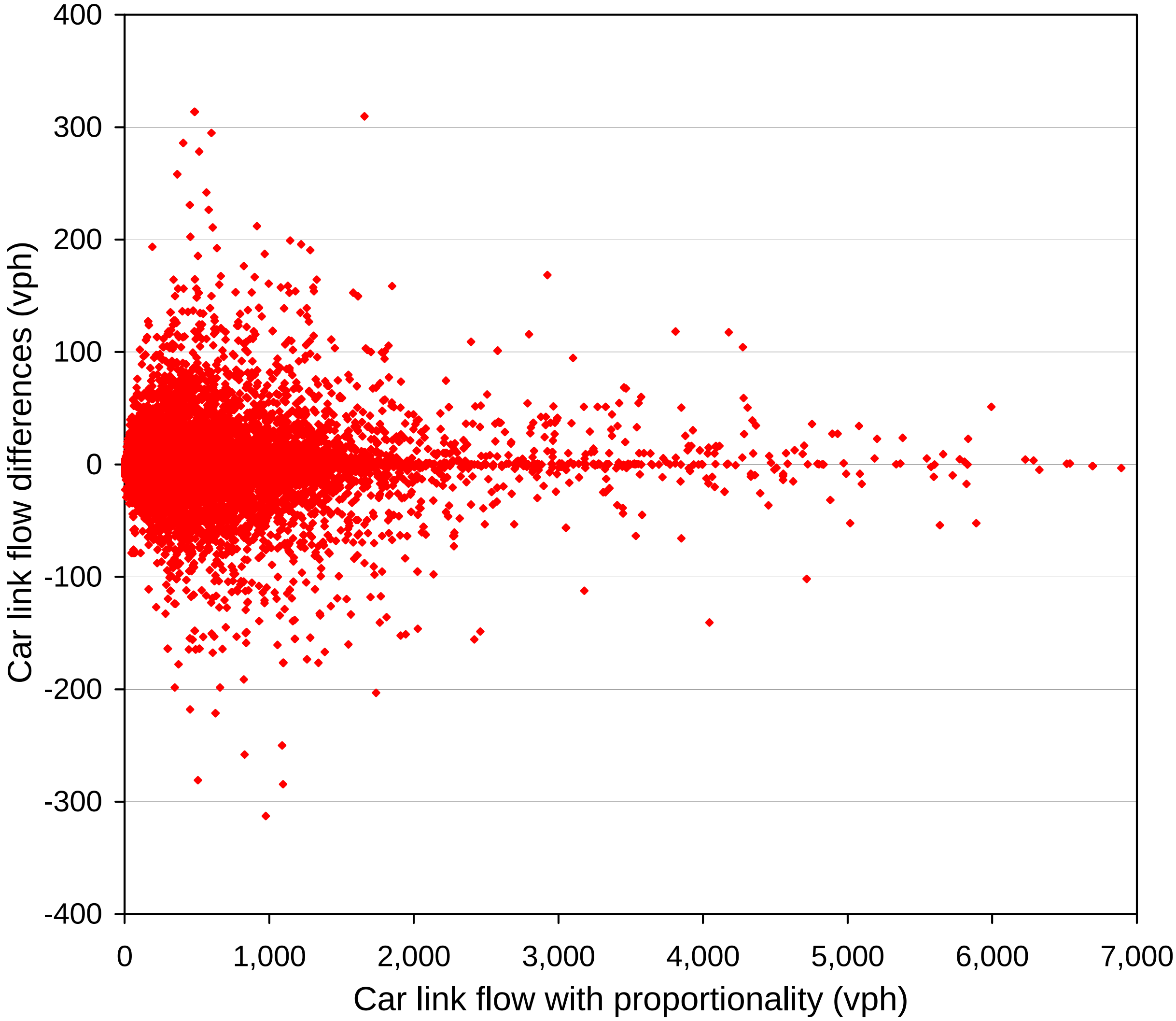


**Fig. 6**. Differences in car link flows versus car link flow with proportionality
(link flow with proportionality less link flow without proportionality)

The Chicago regional network consists of 35,436 links, excluding 3,582 centroid connectors. For links that may be used by trucks as well as cars, 19,743 links (57%) have zero difference in class link flows with and without proportionality. Of these links with zero flow difference, 4,509 links (23%) are not used by either class, and are primarily located at the periphery of the Chicago region; thus, 15,243 used links have zero flow differences. Zero class flow differences suggest these links may not be included in any pair of alternative segments; in this case their class flows are uniquely determined. Also excluded from the analysis are 562 car only links.

The number of links with a given difference in car link flow with and without proportionality is shown in Fig. 7a by seven link flow difference intervals. The vertical axis is the logarithm of the number of links in the link flow difference interval. About one-half of all links have no link flow difference, one-third have differences between 0 and +/–10, one sixth have differences between +/–10 and +/–100, and nearly 300 have differences exceeding +/–100. The histogram for truck link flow differences (not shown) is the exact mirror image of the car flow histogram.

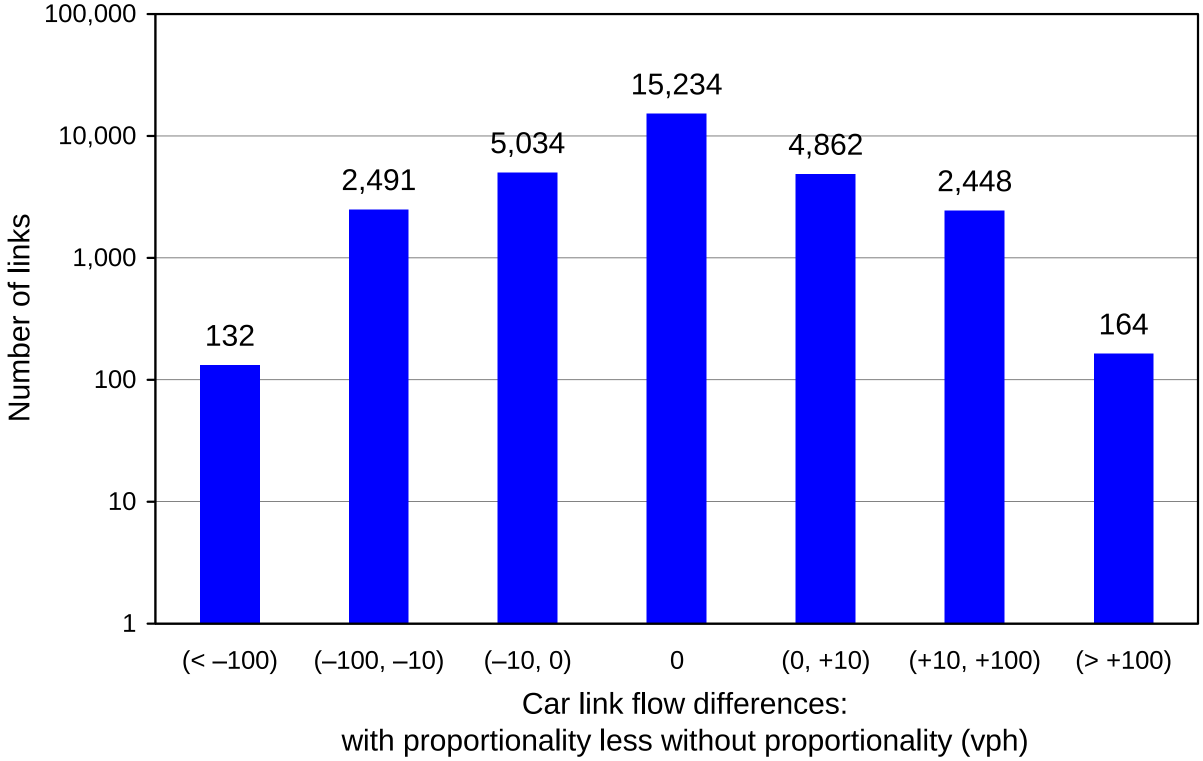


**Fig. 7a.** Number of links by car link flow difference categories

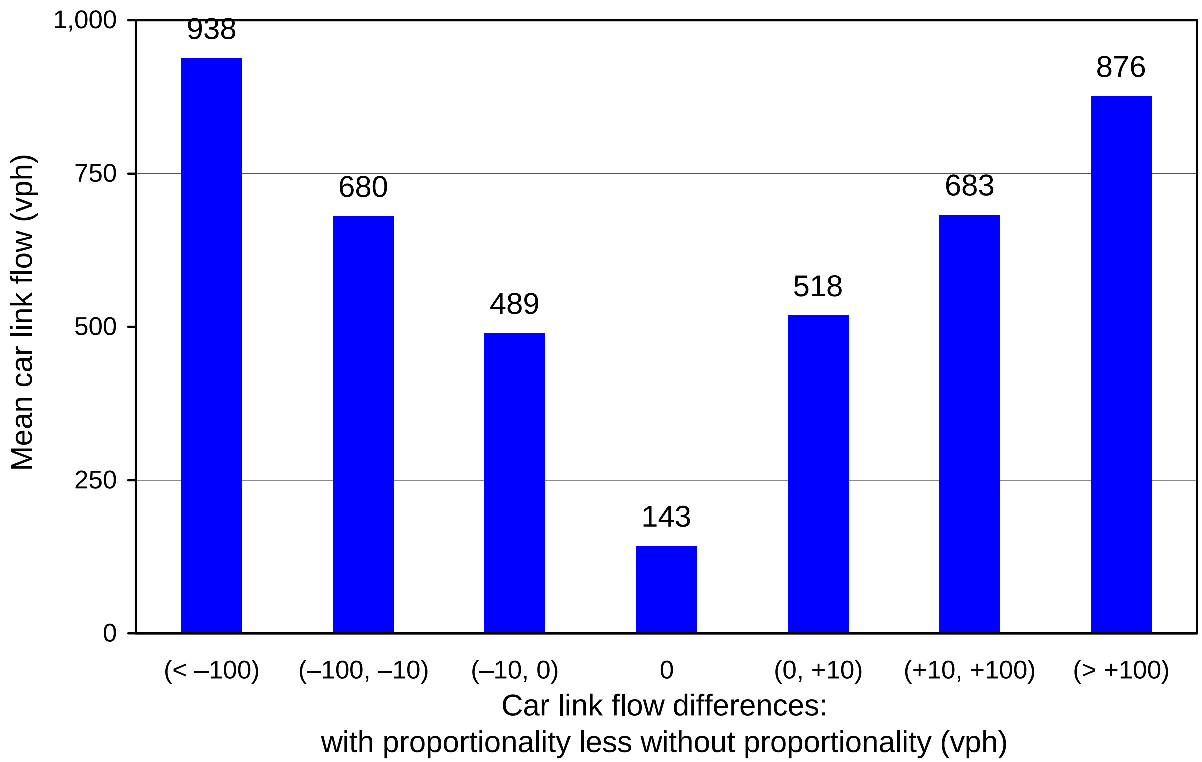


**Fig. 7b.** Mean car link flow by car link flow difference categories

Fig. 7b shows the mean car link flows for the same intervals defined above. For example, the 132 links with car link flow differences smaller than –100 have average car flows of 938 vph. In contrast, the 15,243 used links with zero car flow difference have an average flow of 143 vph.

Car link flow differences were also plotted versus the link volume-to-capacity ratio and link congested speed links in an exploratory analysis (not shown). The link flow differences are unimodally distributed with respect to volume-to-capacity ratio and clustered by congested speed, reflecting the frequency distribution of links by free flow speed. No interesting insights or anomalies were noted.

As shown in Section 4, the arbitrary choice of initially assigning the car matrix before the truck matrix resulted in substantially different class segment flows from the opposite choice of assigning truck before car. Since the car and truck flows between each O-D pair are unchanged by the order of assigning the trip matrices, and since the total link flows are uniquely determined by each assignment, the two total link flow solutions should be equal, subject to any differences in convergence. Comparison of the total link flows for 30,365 links (excluding centroid connectors, links with zero flow in both solutions and car only links) showed very small differences in flow for 16,366 links (54% of all links with flow). The total absolute flow difference (TAFD) is 629 vph; the root mean square difference (RMSD) is 0.26. These measures of difference in convergence provide a basis for comparing differences in class link flows between the two assignments.

To compare and visualize these results, car link flows from the two assignments before adjustment for proportionality are plotted in Fig. 8a. The TAFD for cars is 337,800 vph, or 537 times the difference in convergence stated above; the RMSD is 33.2, or 130 times the difference in convergence. These differences reflect the effects of the order of the trip matrices and the non-uniqueness of the class link flow solutions, as well as differences in convergence between the two assignments. The maximum difference in link flows between the two solutions is shown at the top of each chart.

Fig. 8b shows the differences in the car link flows between the two assignments after adjustment for proportionality. The TAFD between the two adjusted assignments is reduced to 35,700 vph, or 57 times the difference in total link flow convergence; the RMSD is 4.5 or 17 times the difference in total link flow convergence. The differences reflect the overall convergence of two assignments and deviations from proportionality following the post-processing adjustment of each assignment. Computational procedures currently available in Visum do not permit more precise convergence. Further refinement of computational precision is an ongoing challenge for software developers.

Travel forecasting software developers generally realize that the specification of the order of the trip matrices in an assignment affects the class link flows; however, this artifact may not be widely appreciated by practitioners. Similar effects might be introduced by renumbering zones or by large-scale network coding changes associated with scenario analyses. Practitioners should be aware that such changes could result in differences in class link flows that may not be attributed to differences in scenario inputs.

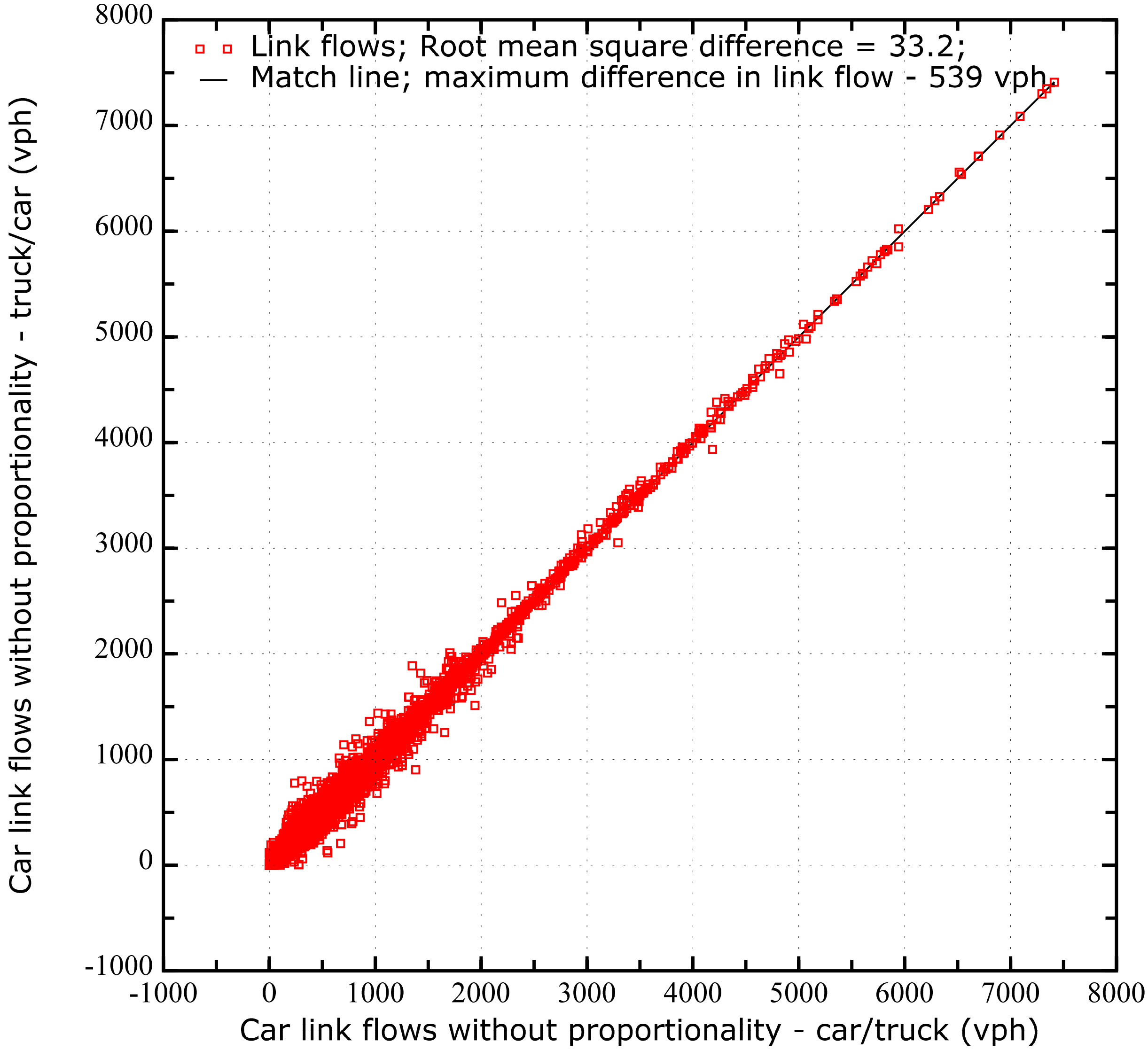


**Fig. 8a.** Car link flows without proportionality: truck/car versus car/truck.

## 6. Class O-D Flows on a Selected Link

Select link analyses are frequently performed by practitioners to explore the attributes of their assignments and to address questions of interest to decision makers. In this section the effects of imposing proportionality are examined at the level of O-D flows on an individual link. Similar results were found for truck flows, but are not shown.

Figs. 9a and 9b show the results of select link analyses for the North Avenue bridge in the eastbound direction located two miles north of the Central Area (link 6380-6389 in Fig. 3) for the car/truck and truck/car assignments described in Section 4. North Avenue is a four-lane, two-way arterial with a directional capacity of 1,540 vph. Each square represents one O-D flow. The O-D flows without proportionality are shown on the vertical axis, while the O-D flows with proportionality are shown on the horizontal axis. The match line indicates O-D pairs which have the same flow with and without proportionality.

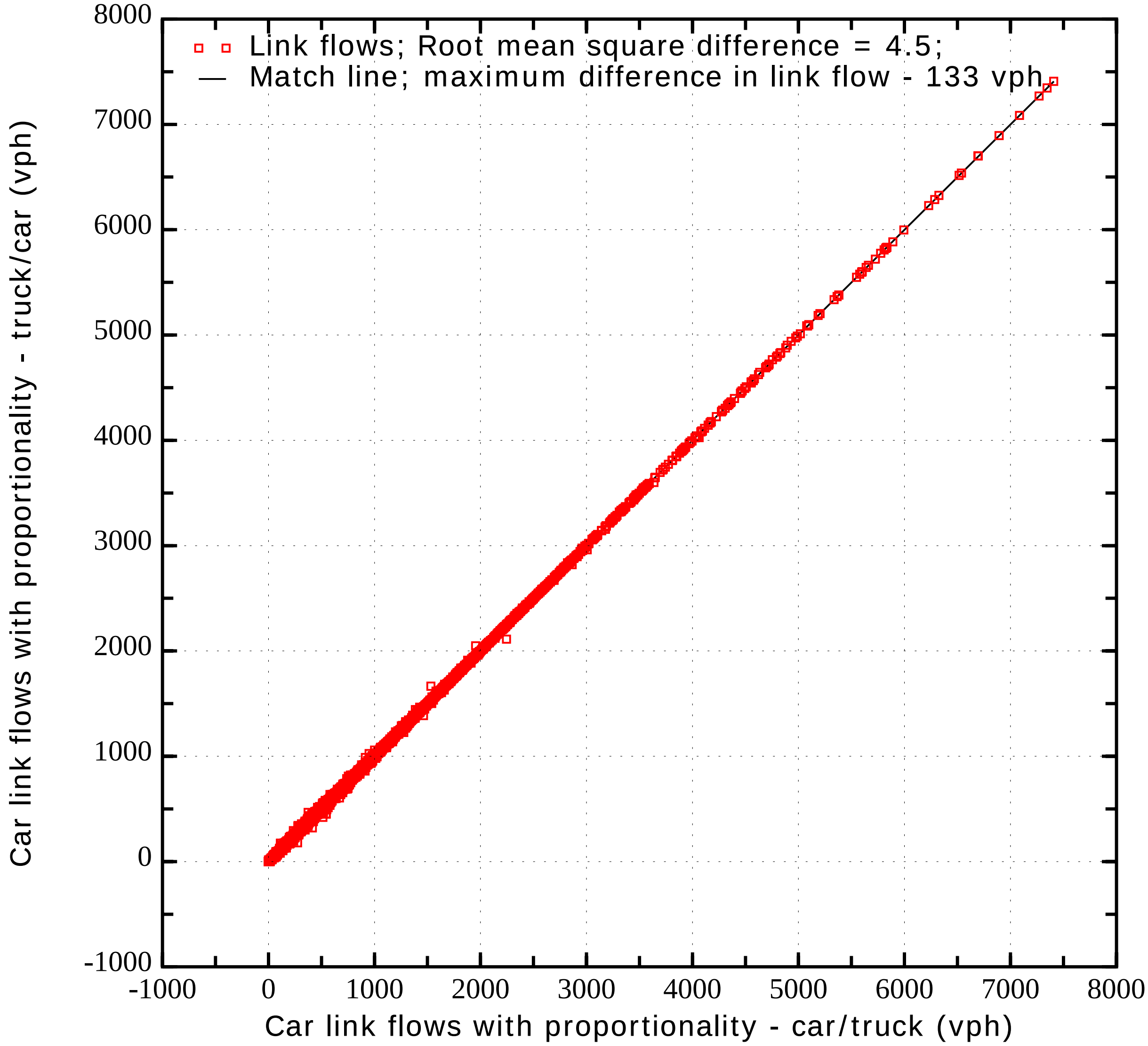


**Fig. 8b.** Car link flows with proportionality: truck/car versus car/truck.

Figs. 9a and 9b compare car flows for the car/truck and truck/car assignments. Several O-D pairs have identical flows over the North Avenue bridge with and without proportionality, as shown by the points on the match line; these O-D flows do not cross a pair of alternative segments, and hence have only one UE route available. Fig. 9a shows two groups of O-D pairs for which the flows from the two assignments are different. For some 30 O-D pairs, whose points lie above the match line, the flows without proportionality are higher than the flows with proportionality by as much as 50%. Another group of small O-D flows lie along the horizontal axis between 0 and 4. These O-D pairs have no route flows over the bridge in the assignment without proportionality. A similar pattern may be observed in Fig. 9b. Note the number of O-D pairs in each assignment on each axis of Fig. 9a: 2,576 without proportionality vs. 2,780 with proportionality. Therefore, small flows between 204 O-D pairs were added in Fig. 9a by the proportionality adjustment; 128 O-D pairs were added in Fig. 9b.

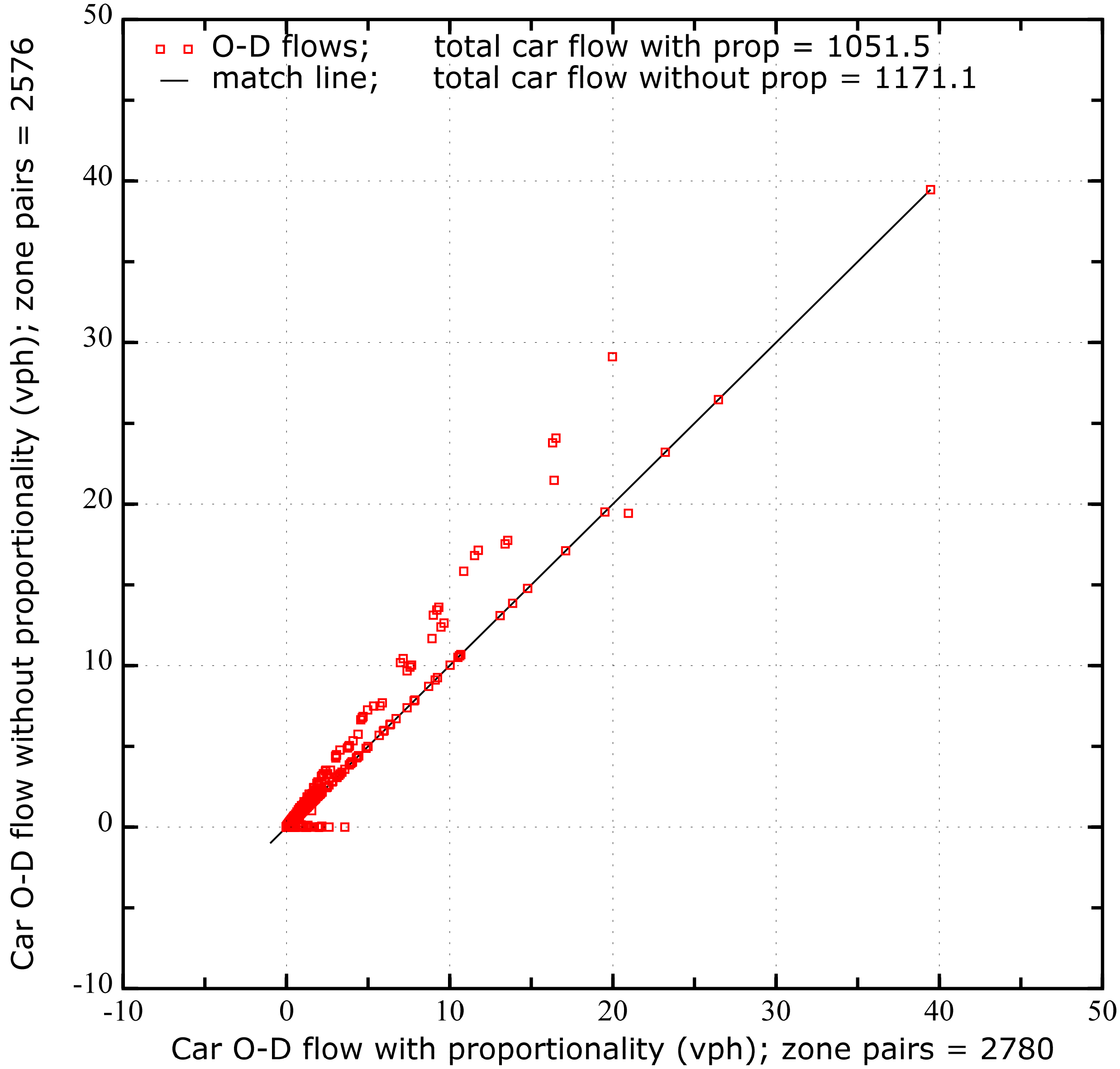


**Fig. 9a.** Car O-D flows: order - car/truck.

Next examine the total car link flows with and without proportionality shown at the top of Figs. 9a and 9b. The values of total car link flow with proportionality are 1051.5 vph and 1050.2 vph; the small discrepancy between these two total flows is a result of minor differences in convergence towards the UE solution. In the car/truck assignment, the total car link flow without proportionality is 120 vph higher, and in the truck/car assignment, the total car link flow without proportionality is 79 vph lower, than the total flow with proportionality. Therefore, the overall discrepancy in car link flows without proportionality between the two assignments is 199 vph or 19% of the total flow with proportionality.

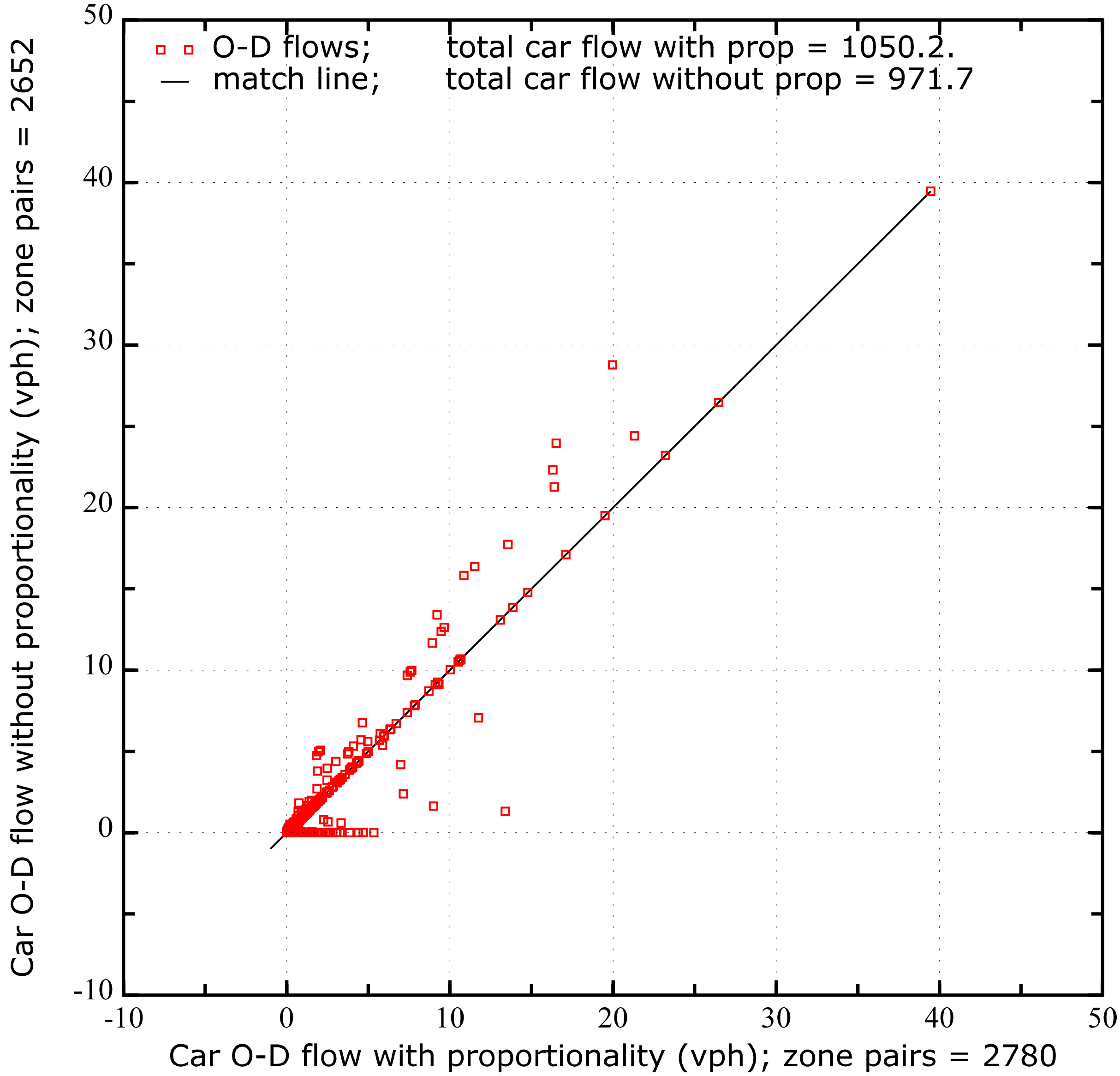


**Fig. 9b.** Car O-D flows: order - truck/car.

Select link analysis is widely used by practitioners to display and interpret the results of an assignment of interest to decision makers. This example for an arterial of moderate capacity shows three attributes of the use of this method for multi-class assignments: 1) the order in which the trip matrices are specified may substantially change the route flow results, as well as the class link flow results, as noted previously in Section 5; 2) the route flows after adjusting for proportionality may differ substantially from the unadjusted flows obtained directly from an assignment; 3) the number of O-D pairs using a link after adjustment may differ from the unadjusted flows. For all of these reasons, the use of a post-processing proportionality adjustment is recommended if flows are presented and interpreted at a more detailed level than total link flows.

## 7. Conclusions

The effects of imposing the condition of proportionality on car and truck link flows from two assignments on a large-scale road network of the Chicago region were compared at three levels: 1) class flows over a pair of alternative segments; 2) car link flows over the entire network; 3) car O-D flows over a selected link. The effects of the ordering of the trip matrices in the assignment on the resulting class link flows and class O-D flows without and with the condition of proportionality imposed were examined with the aid of tables and charts.

Differences in class link flows without and with proportionality range up to +/– 300 vph. Relative differences in segment class flows for the case examined are more substantial, the class flows on one segment being more than three times as large after adjustment for proportionality. Differences in class flows over a selected link are as high as 50% for large O-D flows; moreover, many small O-D flows are omitted from the assignment without proportionality.

Such artifacts arising from the ordering of trip matrices are one example of how arbitrary coding and software setup decisions may affect assignment results. Whether these differences are large enough to warrant concern in applications is a matter for practitioners to judge. To assure that class link flows are uniquely determined, especially in the case of special purpose facilities, such as truck routes, practitioners are advised to impose proportionality on their multi-class traffic assignments, especially if the same generalized cost function is used for each class.

Class link flows without proportionality are partially determined by the ordering of the class trip matrices. The designation of one matrix as ‘primary’ by some practitioners, or their software, may suggest that one order is more appropriate than another; in fact, such order-related differences arise arbitrarily. In some software systems the order of assignment matrices may be determined internally by undisclosed criteria. These procedures may also lead to arbitrary results.

The imposition of the condition of proportionality has the potential to improve the usefulness of traffic assignments in transportation planning practice by determining both route flows and multi-class link flows uniquely. At the time this paper was prepared, two travel forecasting software systems (TransCAD and Visum) offered a capability to impose proportionality through post-processing of bush-based assignment solutions.

Additional research is needed to understand the effects of assuming a different generalized cost function, or different parameter values, for each class. The use of different functions may cause class link flows to be uniquely determined. Research in progress is seeking to determine to what extent proportional route and class link flows occur in reality. The findings of this research could offer an additional basis for justifying the imposition of the condition of proportionality.

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