An Exploratory Analysis of PAS Characteristics in Solving the Static Deterministic User Equilibrium Traffic Assignment Problem on a Large Scale Urban Network

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Abstract

The standard static deterministic user equilibrium (UE) traffic assignment has long been one of the most intensively applied tools utilized by transportation practitioners particularly for travel forecasting. As is, however, well known, the standard UE formulation provides a unique solution for total link flows but not for route flows. In 2010, Bar-Gera proposed a new algorithm called Traffic Assignment by Paired Alternative Segments (TAPAS) to solve the static deterministic UE traffic assignment in a computationally efficient manner while the first time addressing the issue of route flow uniqueness by incorporating a condition of proportionality.

Although TAPAS has been successfully implemented on various scales of transportation networks, characteristics of pairs of alternative segments (PASs) that comprise the solutions of the standard UE model, especially for a large-scale urban network, have not been adequately explored and revealed. This study examines and compares the underlying characteristics of PAS solutions for the Chicago regional network at both aggregate and disaggregate levels for three congestion scenarios applied to a single-class model. Investigations utilize descriptive analysis as well as graphical and map representations of selected PASs to substantiate observations and conclusions. Findings of this study are expected to help guide transportation practitioners in understanding the properties of traffic assignments with unique route flows and in assessing the performance of algorithms in traffic assignment problems.

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1. Introduction

The standard static deterministic user equilibrium (UE) traffic assignment has long been one of the most intensively applied tools utilized by transportation planning practitioners particularly for travel forecasting. Although it is well known that the standard UE formulation provides a unique solution in terms of total link flows but not for route flows, route flows are widely used in practice at various levels of aggregation (Bar-Gera, 2002, 2010). Currently used applications of route flows are based on an arbitrary choice among possibly an infinite number of route flow solutions, which may lead to inconsistent answers in partially aggregated analyses. Bar-Gera and Luzon (2007) evaluated the appropriateness of arbitrary choices between different UE route flow solutions and suggested a behaviorally justifiable condition to enhance the consistent choice of route flow solutions. In 2010, he proposed a new algorithm called Traffic Assignment by Paired Alternative Segments (TAPAS) to solve the static deterministic UE traffic assignment in a computationally efficient manner while ensuring route flow uniqueness by incorporating a condition of proportionality. The fundamental principle of proportionality states the proportion of travelers on each of the two alternative equal cost segments must be the same, regardless of their origins or their destinations. Boyce et al. (2010) conducted a comparative study to determine the extent to which the UE solutions produced by a number of currently available commercial traffic assignment tools satisfy the condition of proportionality as solved by TAPAS.

Although TAPAS has been successfully implemented on various scales of both test and actual networks, characteristics of the Pairs of Alternative Segments (PASs) that comprise the solutions of the standard UE model, especially for a large-scale transportation network, have not yet been adequately explored and revealed. This study seeks to advance understanding of UE solutions of a realistic large-scale urban network with unique route flows; however the ultimate goals of gaining such profundity are expected to help improve currently available theories as well as stimulate creations of novel methods to solve traffic assignment problems in a more effective manner. The approaches used in this study can be characterized as exploratory and empirical.

The rest of the paper is organized as follows. Section 2 offers formal definitions and fundamental principles of PAS. Section 3 examines and compares the underlying characteristics of PAS solutions on the test network at both aggregate and disaggregate levels for three different congestion scenarios. Section 4 provides the readers integrative perspectives of how PASs have roles in forming UE route patterns. Results presented in section 3 and 4 utilize descriptive analysis as well as graphical and map representations of selected PASs to substantiate observations and conclusions. Summary findings of PAS solutions, applicability of the findings for transportation practitioners, and directions for future research conclude the paper in section 5.

2. Formal Definitions and Fundamental Principles of PAS

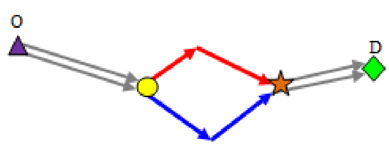


Figure 1: Graphical representation of a PAS with a single OD pair

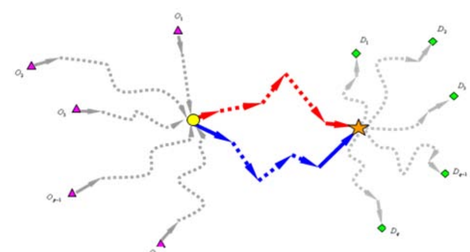


Figure 2: Graphical representation of a PAS with multiple OD pairs

This section offers a formal definition of a pair of alternative segments (PAS) in conjunction with its fundamental principles in an effort to help enhance observations, analyses, and interpretations of PAS solutions in the subsequent sections. A PAS is defined as a pair of the distinguishing (alternative route) segments between a pair of routes. A simple traffic network shown in Figure 1 has two routes connecting one OD pair. These routes only differ by the alternative segments at the middle of routes. Both distinguishing segments are collectively referred to as PAS. The formal definition of PAS can be extended to a more general case, as shown in Figure 2. By definition, every PAS must contain only two distinguishing segments (the red and blue directed links), only one diverge node (a yellow circle), and only one merge node (a brown star). PAS may be relevant to more than one origin (a pink triangle) or destination (a green diamond). Equally important, PAS can not contain any physically identical links.

The fundamental principles of a PAS are deeply ingrained in its conceptual functionalities. In the TAPAS algorithm, a PAS is used for two purposes: cost equilibration by shifting flows within each PAS to ensure the UE route flow solution, and proportionality adjustment by distributing equal cost segment flows between origins to ensure route flow uniqueness. For the former, distinguishing segments for any PAS are not arbitrarily determined but they must be consistent with certain criteria to ensure UE route flow. They are chosen in the way that one segment will consist of used links and the other will be part of the minimum cost tree. Flows are then shifted from the higher to the lower cost segment (i.e., the minimum cost segment), which are conceptually equivalent to shifting flows from the higher to the lower cost route . At the time the generalized costs on both segments are precisely equal, the PAS becomes “active.” As can be seen in Figure 1, an active PAS renders travel costs on both routes minimum and identical, thus ensuring that the UE condition is achieved. A principle of equilibrating cost for PAS with a single OD pair is also valid for a PAS with multiple OD pairs, in which each pair can be considered independently when equilibrating cost. Note that flow shifts are only performed on links that are parts of distinguishing segments. In application, the number of links that are parts of PASs is relatively small as compared with those that are not parts of PASs. Focusing only on PASs makes the flow shift computations of TAPAS much more efficient than the method of Frank-Wolfe in which all links in the network are used to compute a stepsize in the line search process. Although flow shift operations are considered to be the key competitive advantage of the TAPAS algorithm, its primary success derives from its capability to determine PASs for every pair of origin and destination of a traffic network with relatively little computation effort. This task is made possible by using a breadth first search procedure, which usually leads to PASs with small number of links that are additive in the line search for cost equilibration.

For PASs with multiple OD pairs, the segment flows obtained by the cost equilibration process typically yield different proportions of segment flow for different OD pairs. To ensure unique route flows, as well as the standard uniquely determined total link flows, the OD flows for every OD pair must be allocated across these segments. When the proportion of segment flow is equal among OD pairs, the proportionality condition is satisfied. Note that proportionality adjustment is not necessary for PAS with a single OD pair. The complete descriptions of TAPAS algorithm as well as theoretical concepts are elaborated in Bar-Gera (2006, 2010).

3. Characteristics of PAS Solutions

The TAPAS algorithm was utilized to assign three OD trip matrices to the Chicago regional network, which consists of 12,982 nodes, and 39,018 links with 27,050.2 miles of total network length. These OD trip matrices were devised by using a Mode-Origin-Destination (MOD) model, which is a doubly-constrained logit form, $d_{mpq} = A_p B_q \exp(-\mu \cdot c_{mpq})$, where (A_p, B_q) are balancing factors to ensure that the constraints on total originating and terminating flows hold, μ is a cost sensitivity parameter, and c_{mpq} is mode-origin-destination level of service or generalized travel cost. Three OD trip matrices with different values of the cost sensitivity parameter allow one to observe the effects of travel cost and relative importance of congestion on PAS solutions. The values were selected in the manner that the cost sensitivity values were decreased by one-half from the first (0.20) to the second (0.10), and from the second (0.10) to the third solutions (0.05). The largest value of cost sensitivity has the lowest travel generalized cost and least congestion, since travelers are more sensitive to travel cost. The MOD model implemented in this exploratory study is a single-class case since only the mode of auto is considered and truck trips are reduced to passenger-car-equivalents. This study also assumes that the generalized link travel cost depends only on link travel times, given by the conventional BPR function. Interested readers can consult Bar-Gera and Boyce (2005, 2007) for the detailed construction of these three OD trip matrices. In this section, key characteristics of the resulting PAS-based solutions are probed at both aggregate and disaggregate levels. However, interested readers can refer to Bar-Gera and Boyce (2005, 2007) for characteristics of route-based solutions solved by OBA algorithm on the same test network and cost sensitivity values. Note that term PAS used in section 3-5 specifically refers to PAS at equilibrium solutions or active PAS.

3.1 Aggregate Characteristics of PAS Solutions

To help visualize the characteristics of PAS solutions on the entire Chicago regional network, all links of the network are plotted with respect to their geographical locations and functional classes of link solutions for each value of cost sensitivity. As can be seen in Figure 3, three colored links are used to represent each functional class. A green link is a used link that is a part of a PAS. A blue link is a used link that is not part of any PAS. And a red link is an unused link. Supplementary to each value of cost sensitivity, the mean route travel time in minutes, as shown in parentheses, is employed as an overall measure of network congestion to aid readers in interpreting cost sensitivity values. For example, on the heavily congested network, travelers spend about 80 minutes on average traversing from any zone of origin to any zone of destination. To gain complete understandings of PAS characteristics, Figure 3 must be interpreted in conjunction with corresponding information in Tables 1 and 2.

Observations reveal that the number of active PASs and the number of UE routes depend to substantial extent upon the congestion level in the network. As congestion increases, an increasing number of unused links and used links that are not parts of any PAS are superseded by used links that are parts of a PAS. Of 39,018 total network links, about 40-60% or equivalently 32-50% of total network lengths is used and parts of a PAS. About 30-46% or equivalently 31-53% of total network length is used but not part of any PAS. And the remaining 10-12% or equivalently 12-14% of total network length is unused. Total traffic on the network measured by vehicle-miles of travel reveals an interesting characteristic of PAS solutions. Up to 72-86% of traffic is carried by used links that are part of a PAS, whereas only 14-27% of traffic is on used links that are not parts of any PAS.

Overall characteristics of PAS solutions in Table 2 can confirm that the results produced by the TAPAS algorithm favorably comply with fundamental principles of PAS as described in section 2. As can be seen, an active PAS on average has relatively short total link lengths, a relatively small number of links, and a relatively large number of origins. Basically, PASs with shorter total link lengths and less number of links involved are desirable, since they are likely to be relevant to more origins, and a shift of flow between any pairs of routes can be computed faster (Bar-Gera, 2010).

3.2 Disaggregate Characteristics of PAS Solutions

All individual active PASs for each cost sensitivity are investigated to determine whether formal definitions or fundamental principles of PAS are violated. Active PAS can be divided into two main categories: with and without segment crossing(s). However, the first category can be further classified into two subcategories: crossing(s) at a link and crossing(s) at a node. Only segment crossing(s) at a node violate the formal definition of a PAS. PASs with multiple diverge nodes, with multiple merge nodes, or with more than two segments must not be permitted.

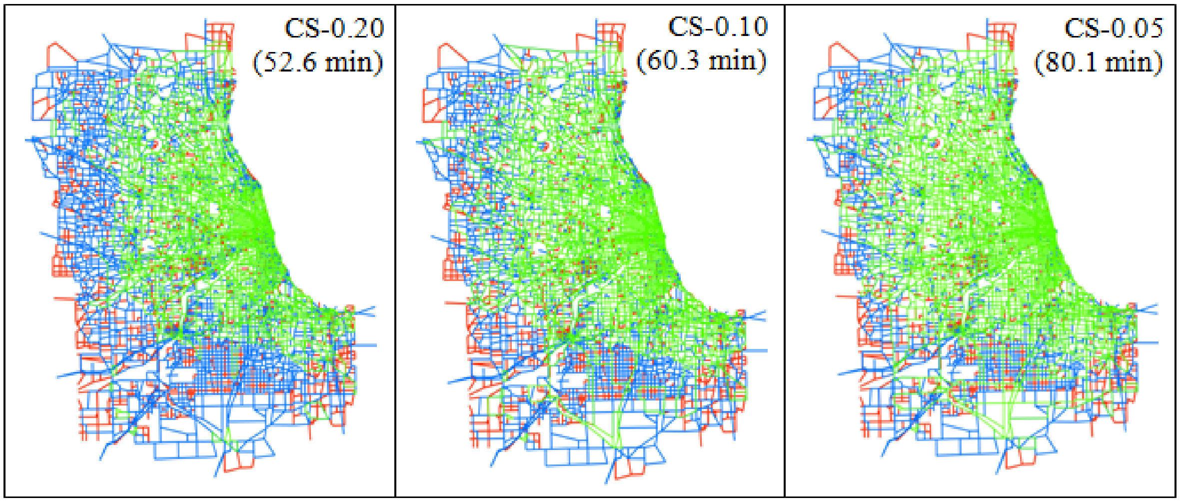


Figure 3: Graphical representation of PAS solutions

Table 1: Detailed PAS solutions classified by geographical locations and functional classes of link solutions

CS	Number of used links that are part(s) of PAS(s) [green links]	Number of used links that are not part(s) of any PAS(s) [blue links]	Number of unused links [red links]	Number of links on a network
0.20	16,270 (41.7%)	18,006 (46.2%)	4,742 (12.2%)	39,018 (100%)
0.10	20,122 (51.6%)	14,609 (37.4%)	4,287 (11.0%)	39,018 (100%)
0.05	23,209 (59.5%)	11,914 (30.5%)	3,895 (10.0%)	39,018 (100%)
	Total lengths of used links that are part(s) of PAS(s) (miles) [green]	Total lengths of used links that are not part(s) of any PAS(s) (miles)[blue]	Total lengths of unused links (miles)[red]	Total lengths of links on a network (miles)
0.20	8,763.9 (32.4%)	14,350.1 (53.1%)	3,936.3 (14.6%)	27,050.2 (100%)
0.10	11,402.8 (42.2%)	11,962.0 (44.2%)	3,685.4 (13.6%)	27,050.2 (100%)
0.05	13,692.7 (50.6%)	9,957.5 (31.7%)	3,400.0 (12.6%)	27,050.2 (100%)
	VMT on used links that are part(s) of PAS [green]	VMT on used links that are not part(s) of any PAS[blue]	VMT on unused links [red]	VMT on a network
0.20	10,211,158.9 (72.3%)	3,902,982.3 (27.7%)	0 (0%)	3,902,982.3 (100%)
0.10	13,664,006.6 (80.5%)	3,313,350.4 (19.5%)	0 (0%)	3,313,350.4 (100%)
0.05	17,196,053.4 (86.0%)	2,809,233.9 (14.0%)	0 (0%)	2,809,233.9 (100%)

Table 2: Overall characteristics of PAS solutions

PAS Characteristics	CS-0.20	CS-0.10	CS-0.05
Total number of PASs	5,617	11,702	22,500
Average number of links per PAS	19.9	27.1	35.5
Average length per PAS (miles)	10.2	13.2	18.1
Average number of origins per PAS	105.2	100.6	103.3
Total number of UE routes	8,397,772	19,121,834	198,087,738

Fortunately, none has segment crossing(s) at a node, assuring that current mechanism of TAPAS behaves properly. Violations to the fundamental principles of PAS can be detected by searching for physically identical links. Active PAS with physically identical links result in a contradiction to the fundamental assumption of proportionality. Under this condition, the flow proportionality on the two alternative segments of any active PAS with the same physical links must be precisely equal for every OD pair using the PAS. Fortunately, all active PASs of each of the three solutions are physically different, enabling one to determine that current research code of TAPAS does produce accurate solutions that perfectly satisfy proportionality.

To help observe the full range of PAS solutions at disaggregate level, individual PASs for the three values of cost sensitivity are plotted according their physical characteristics, including number of links per PAS, total link lengths per PAS, and number of origins per PAS. Corresponding plots are shown in Figures 4-6. Figure 4 presents the frequency of PASs by the number of links per PAS. As can be seen, the frequency for all three solutions is rather fluctuating and seems not able to provide much insight, especially for larger numbers of links. To reduce the effect of this fluctuation on the graphical representation, a corresponding inverted log cumulative distribution is incorporated. Basic descriptive statistics for the three solutions can be read directly from the figure: the 0.20 solution has a mode of 4, median of 14, and maximum of 147; the 0.10 solution has the same mode of 4, median of 20, and maximum of 148; and the 0.05 solution has the same mode of 4, median of 28, and maximum of 176. About 50% of PASs have a relatively small number of links, roughly less than 30 links per PAS. PASs with very simple structures formed by three or four links are pervasive. PASs with extremely complex structures formed by more than 120 links occur rarely, accounting for less than 1%. Moreover, the cumulative distributions show PASs depend to a substantial extent on the level of congestion in the network, with the intermediate sensitivity solution (0.10) lying between the smaller and larger values.

Figure 5 shows the frequency of PASs by total link length. Aggregate scale with equal intervals on the x-axis is used to show the full range of total link length on one chart. For example, the value 1 represents total link length from 0 to 1 mile inclusive. The value 2 represents total link length from 1 to 2 miles inclusive, and so on. Descriptive statistics of the three solutions are summarized as follows: the 0.20 solution has a mode of 3, median of 7, and maximum of 103; the 0.10 solution has the same mode of 3, median of 10, and maximum of 169; and the 0.05 solution has the same mode of 3, median of 13, and maximum of 133. One can see that PASs which are short and local are commonplace. About 50% of PASs have total link lengths less than 13 miles, which is relatively short as compared with the maximum total link length. Occurrences of PASs which are extremely long and regional are rare. PASs with total link lengths greater than 100 miles occur in less than 0.3% of the cases. Similar to Figure 4, the cumulative distributions reveal that PASs are dependent largely on congestion levels with the exception of the extremely long length, in which the maximum cost sensitivity solution lies between the minimum and intermediate cost sensitivity solutions.

Figure 6 gives the frequency of PASs by number of relevant origins in a PAS. The 0.20 solution has a mode of 1, median of 9, and maximum of 1,781. The 0.10 solution has the same mode of 1, the same median of 9, and the same maximum of 1,781. And the last solution has the same mode of 1, median of 17, and maximum of 1,775. One can see that PASs with one origin occur most frequently: 50% of PASs have a relatively small number of relevant origins as compared with the maximum values. Another 50% have moderate to high number of relevant origins, which are very desirable for cost equilibration process.

Figure 4: Number of PASs versus Number of Links per PAS

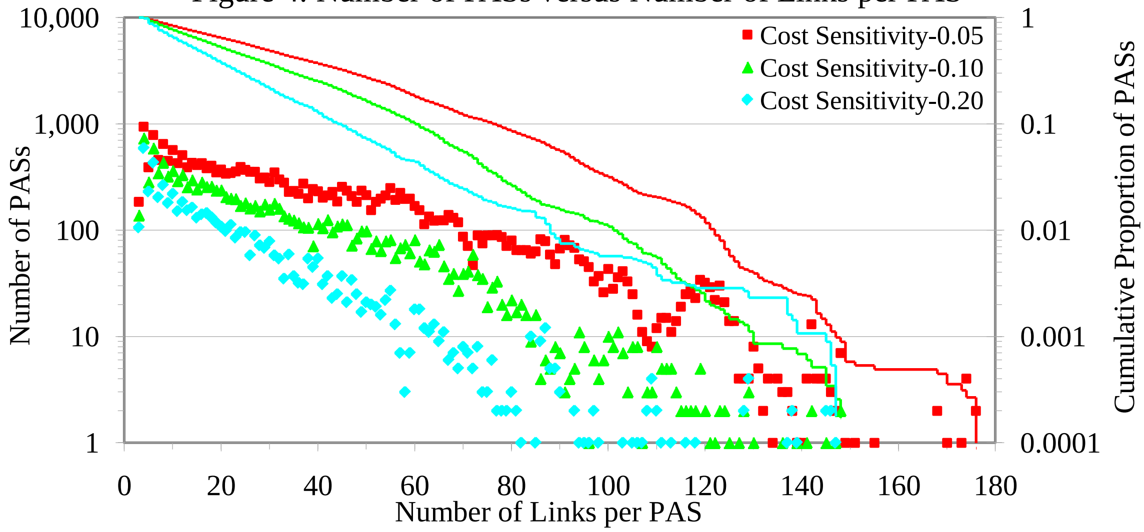


Figure 5: Number of PASs versus Total Link Length of PAS

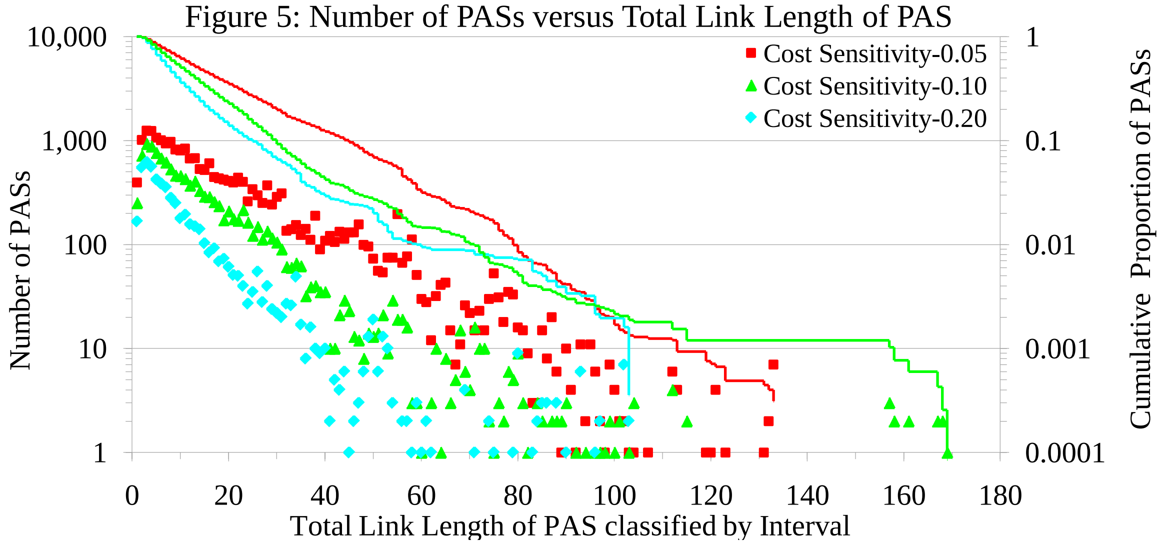
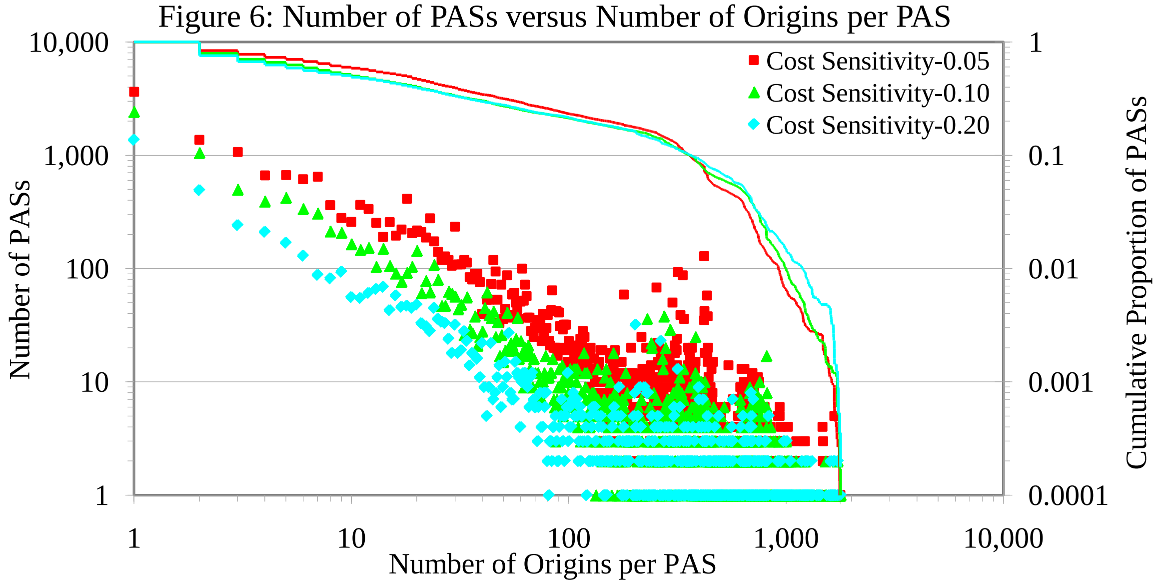


Figure 6: Number of PASs versus Number of Origins per PAS



One may be surprised to see a few PASs with extremely large number of relevant origins, which include almost all origins in the network. By examining the solution in closer detail, it is found that PASs with extremely high number of origins are relevant to very few destinations. Lastly, one may observe that patterns of PAS solutions compared between three values of cost sensitivity parameter look very similar in shape. These indicate that the solutions revealed by the TAPAS algorithm are fairly stable.

4. Contributions of PASs in forming UE route patterns

The central emphasis of this section is to provide an integrative perspective and understanding of how PASs can play roles in forming UE route patterns. Graphical representations of UE route patterns for a selected origin and destination are utilized to observe this contribution, which is illustrated at two levels of analysis: aggregation by an origin, and disaggregation by an OD pair.

In Figure 7, all PASs (the red lines) relevant to a specific origin (a light blue triangle) are plotted over of a tree of UE travel cost routes (the black lines), showing all route flows that emanate from this same origin and terminate at all possible destinations (a green circle). The brown background indicates the links unused by a specific origin. The UE route pattern from origin 1608, which has the maximum number of relevant active PASs in the 0.20 solution, is chosen for this analysis. Although on statistical basis the UE route pattern from only one origin does not necessarily represent all other origins in this solution, it at least does offer some idea of what might occur in other cases. To help determine the relative contributions of PASs across three levels of congestion, maps of the UE route patterns from the same origin for the 0.10 and 0.05 solutions are included as shown in the center and right of Figure 7. Key attributes related to origin 1608, which are placed on the upper right of each map, includes the number of PASs, the number of UE routes, the number of UE routes in which there is only one UE route for each OD pair, the originating flow in vehicles per hour, and mean route travel time in minutes. Decreasing originating flows seem unexpected. However, one should recall that as congestion increases changes in MOD flows could be positive, negative, or even constant, which depend upon the interactions of two interrelated factors: a decreasing value of cost sensitivity and the extent to which the route travel time is increased. Mean route travel times, which are increased as expected, are used as the congestion measure of a network in which all route flows are emanated from origin 1608. It can also be seen from each map in Figure 7 that all PASs relevant to origin 1608 are parts of a tree of UE travel cost routes, which is consistent with fundamental principles of PAS as described in section 2.

The number of PASs and number of UE routes aggregated by an origin depend largely on the congestion level. The number of UE routes increases mainly as a result of the increasing number of PASs. However, one can determine the extent to which PASs contribute to form UE route patterns by taking the ratio of the number of UE routes to the number of PASs. For origin 1608, one PAS on average makes up 6.6 UE routes for the 0.20 solution, 7.1 UE routes for the 0.10 solution, and as many as 32.9 UE routes for the 0.05 solution. It is of interesting to note that with the increasing congestion, the number of OD pairs for which previously there is only one UE route decreases due to the more alternative route segments between each OD pair. The number of single UE routes aggregated by origin 1608 comprise about 25-48% of the total routes in a tree of minimum travel cost route. Finally, graphical representations in Figure 7 unveil spatial arrangements of PASs in a network. As can be seen, PASs are typically narrow in shape and physically aligned with the direction of destinations. PASs with long total link lengths are mostly found at outer suburbs of the region, meanwhile short PASs are mainly in inner suburbs.

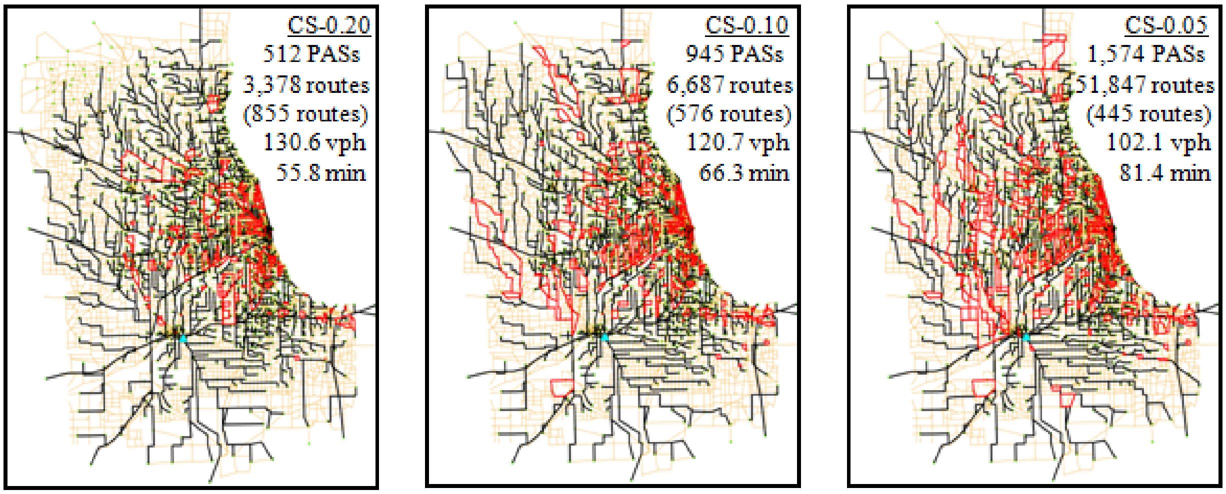


Figure 7: UE route patterns from Origin 1608 which has the maximum number of PAS in the 0.20 solution

A similar approach as applied to aggregation by an origin is used to gain an understanding of UE route patterns disaggregated by an OD pair. PASs (red lines) relevant to a selected OD pair are plotted over the UE routes (blue lines) connecting the same OD pair. Links unused by a selected OD pair (brown lines) represent the background. Figure 8 shows the UE route patterns of the three solutions for OD pair 300-1480, which has the maximum total link length of a PAS in the 0.05 solution. Inside the information box on the top of each map are the number of PASs, the number of UE routes, the OD flow, mean route travel cost in minutes, mean route travel distance in miles, and mean travel speed in miles per hour. Since a high route travel time is chiefly caused by long travel distance, mean travel speed is used as a congestion measure for a specific OD pair. A decrease in travel speed is primarily the result of an increase in route travel cost. A relatively large increase in demand for travel between OD pair 300-1480 with a relatively small increase in route travel cost indicates that this OD pair is fairly sensitive to travel cost. By concept, UE routes between OD pairs may have no, one, or more PASs. On the left map where traffic is fairly uncongested, no alternative route segment is observed. Hence, there is only a single UE route for this OD pair. On the center map where traffic is moderately congested, the UE route is shifted to the north. However, to avoid congested segments of a route, 1 PAS near the zone of origin and 4 PASs near the middle of a route are discovered and used, resulting in total of 6 UE routes. On the right map, where traffic is heavily congested, 29 PASs result in 16 UE routes. 14 UE routes come from 28 PASs that are mostly short and local. The remaining two UE routes are derived from 1 PAS with an extremely long total link length. It is also of interesting to note that almost of all used links in the 0.05 solution are part of PASs. But this does not necessarily mean that PASs in the 0.05 solution have a greater contributions in forming UE route patterns than those in the 0.20 and 0.10 solution. The ratio of the number of PASs to the number of UE routes is used as a measure to determine level of contributions. For the specific OD pair 300-1480, one UE route on average uses 0, 0.83, and 1.81 PASs respectively.

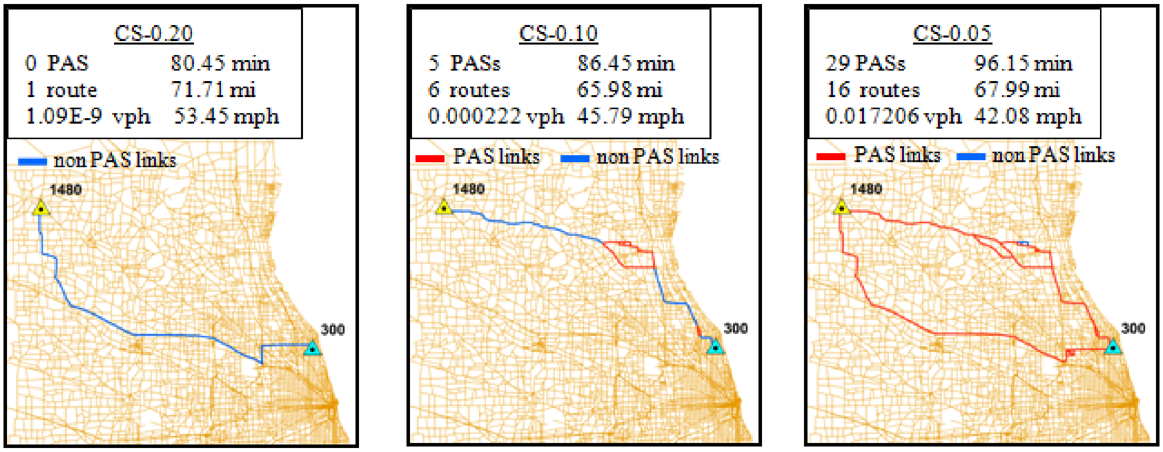


Figure 8: UE route patterns for OD Pair 300-1480 which has the maximum total link length of PASs in the 0.05 solution

5. Summary Findings of PAS Solutions and Future Research

This exploratory research provides for the first time a solid basis for route flow validation of the TAPAS algorithm. It suggests that current research code of TAPAS produced solutions that not only perfectly satisfy the formal definitions of PASs but also strictly comply with the underlying principles of PASs. The following key findings of PAS solutions confirm prior statement: (1) all active PASs must consist of physically unique links; (2) no active PASs has a segment crossing at a node; (3) simple active PASs formed by three or four links are prevalent; (4) active PASs which are short and local are commonplace; (5) occurrences of active PASs which are extremely long are rare; (6) active PASs with small numbers of origins occur most frequently; and (7) all active PASs relevant to a specific origin are part(s) of a corresponding tree of minimum travel cost routes.

Although the findings of this study may not yield instant benefits to transportation practitioners, they can help guide them in understanding the properties of traffic assignments with unique route flows and in assessing the performance of traffic assignment algorithms. Recently, the issues of consistent route flow solution and quick precision have become increasingly important in practical applications. TAPAS exhibits superior computational efficiency which is highly suitable for practical large scale traffic networks and demonstrates a promise of incorporating proportionality into a full feature commercial software. With extensive testing for validity, the results produced by TAPAS provide transportation practitioners a solid basis for their analyses.

Finally, potential subjects for future research could be directed in five main areas. The most immediate topic is to extend the current analysis method to include the behavioral components of active PASs. Investigating characteristics of PAS solutions with more complete generalized link cost functions, such as a linear combination of travel time, tolls, and travel distance, is the one next step that should be pursued. Another interesting area is to replicate analyses with transportation networks from other regions that are spatially different than the currently implemented network. Targeted spatial arrangements include the networks with weaker or stronger grid like, ring or web, radial, or irregular patterns. Experimenting with different trip matrices especially ones with the extreme congestion is perhaps a promising area. Finally, the incorporation of multiple classes into traffic assignment problems should be considered.

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