

# An Exploratory Analysis of PAS Characteristics in Solving the Static Deterministic User Equilibrium Traffic Assignment Problem on a Large Scale Urban Network

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# Overview

- Motivation
- Proportionality condition
- General structure of TAPAS algorithm
- Fundamental principles of Paired Alternative Segments
- Basic characteristics of a test network
- Solution characteristics of PASs
  - at aggregated levels
  - at disaggregated levels
- Contributions of PASs to forming equilibrium route patterns
- Summary findings of PAS solutions

# Motivation

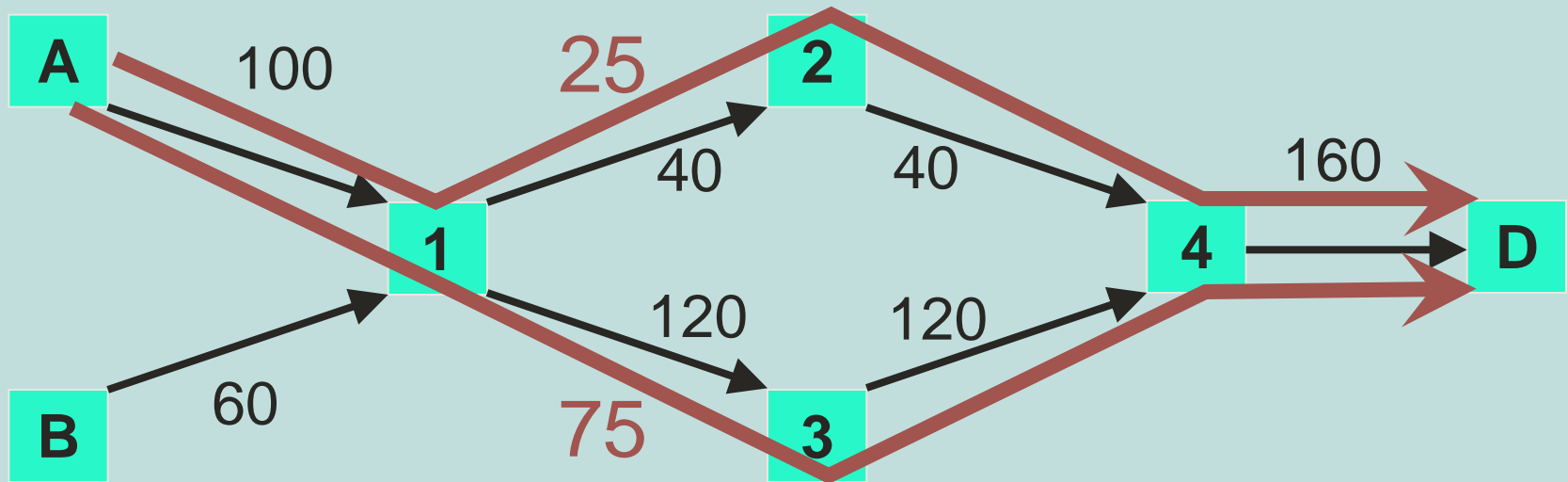
- Static deterministic user equilibrium (UE) traffic assignment has long been one of the most intensively applied tools utilized by transportation planners.
- The standard UE formulation provides a unique solution for total link flows, but not for route flows. However, route flows are often used in practice at various levels of aggregation.
- Bar-Gera (2010) proposed a new algorithm, Traffic Assignment by Paired Alternative Segments (TAPAS) to solve UE traffic assignments efficiently, while determining route flows uniquely by adding a condition of proportionality.
- Determining Pairs of Alternative Segments (PAS) is the key to the success of TAPAS. Focusing on PASs leads to fast convergence, consistent route sets, and unique route flows.
- Although successfully implemented at various scales, several characteristics of PASs, especially for large-scale urban road networks, have not been adequately explored and revealed.

## Proportionality Condition

Same proportions apply to all travelers facing a choice between a pair of alternative segments

Consider the pair of segments [1,2,4] and [1,3,4].

AD segment proportions should be  $1/4$  and  $3/4$ .

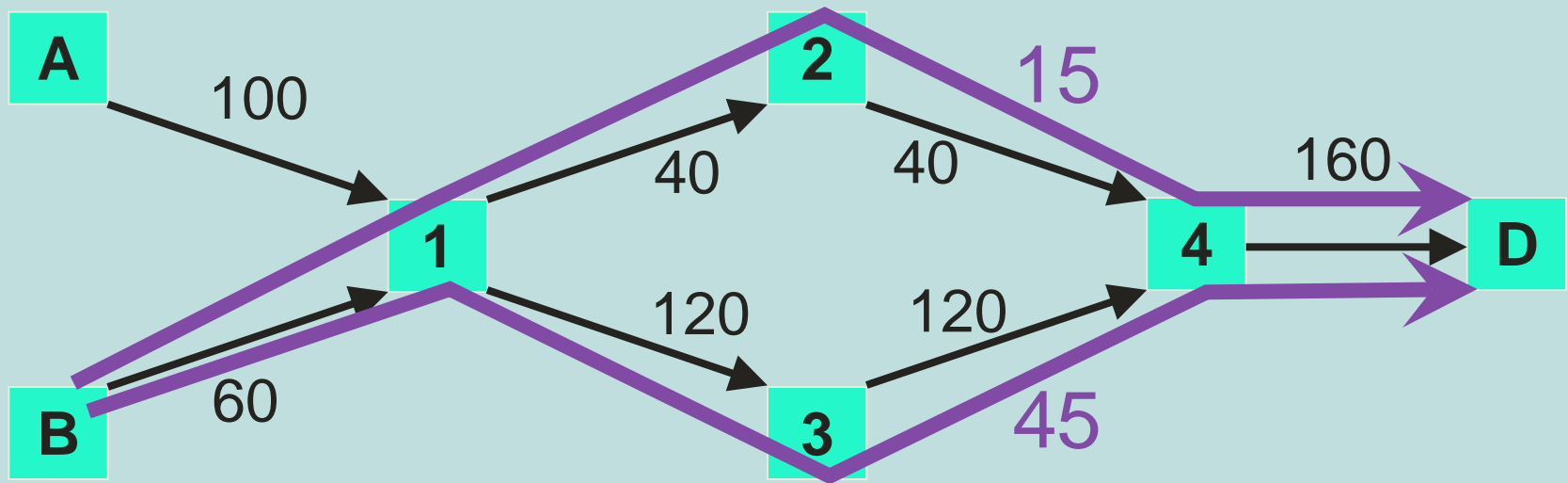


For travelers from A to D the proportion is  $25/(25+75)=1/4$ .

## Proportionality Condition

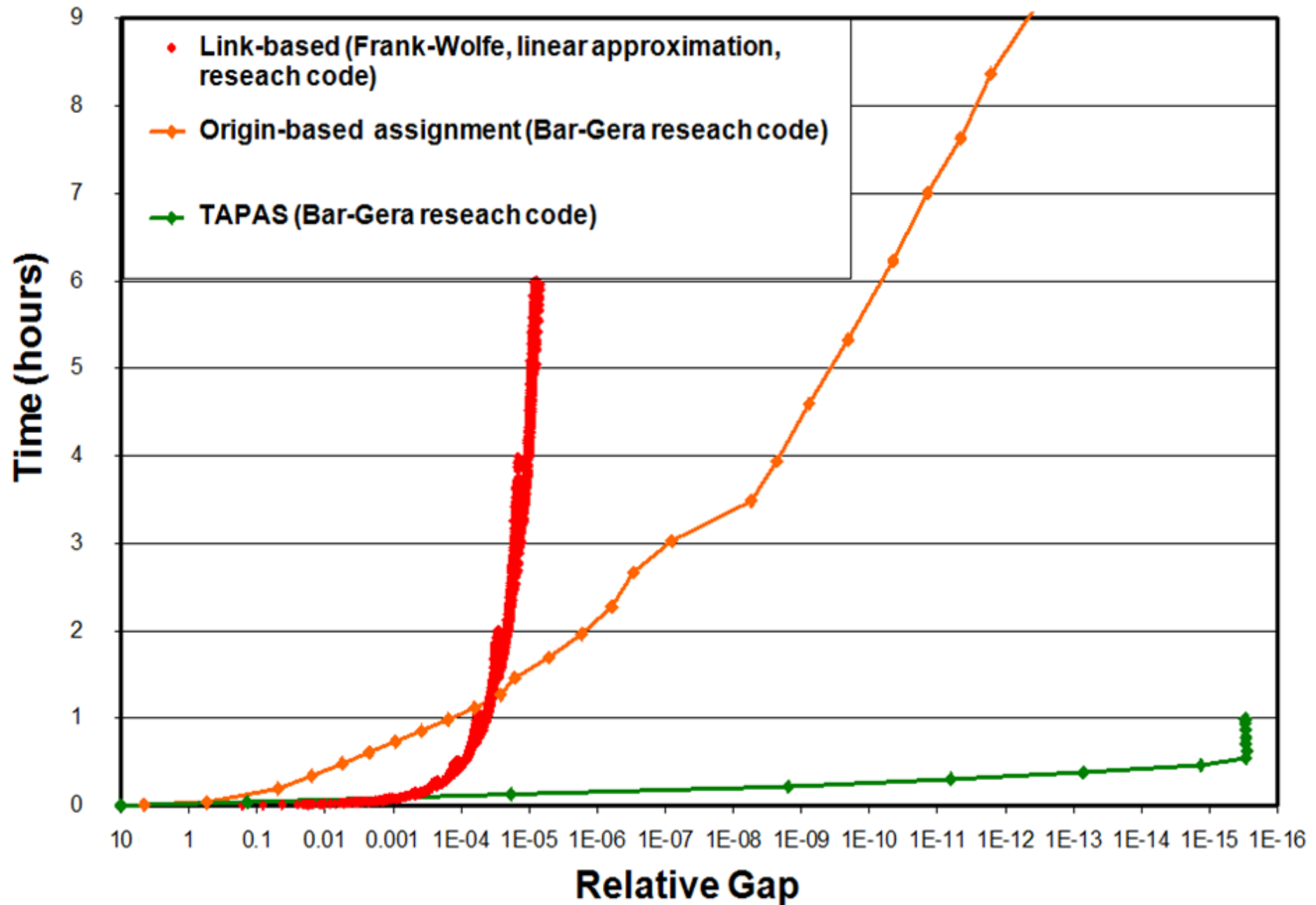
Same proportions apply to all travelers facing a choice between a pair of alternative segments

Consider the pair of segments [1,2,4] and [1,3,4].  
BD segment proportions should **also** be  $1/4$  and  $3/4$ .



For travelers from B to D the proportion is  $15/(15+45)=1/4$ .

# Algorithm Convergence on the Chicago Regional Network



# The general structure of TAPAS algorithm

Find initial solution using AON assignment

Repeat iteratively:

For every origin

Remove all cyclic flows

Find tree of least cost routes

For every link used by the origin which is not part of the tree

If there is an existing effective PAS

Make sure the origin is listed as relevant

Else

Construct a new PAS

Choose a random subset of active PASs

Shift flow within each chosen PAS

For every active PAS

Check if it should be eliminated

Perform flow shift to equilibrate costs

Redistribute flows between origins by the assumption of proportionality

Final proportionality iterations:

For every active PAS

Redistribute flows between origins by the assumption of proportionality

Two functions of PASs:

1) Cost Equilibration

→ Flow shift within each PAS

→ UE route flow solutions

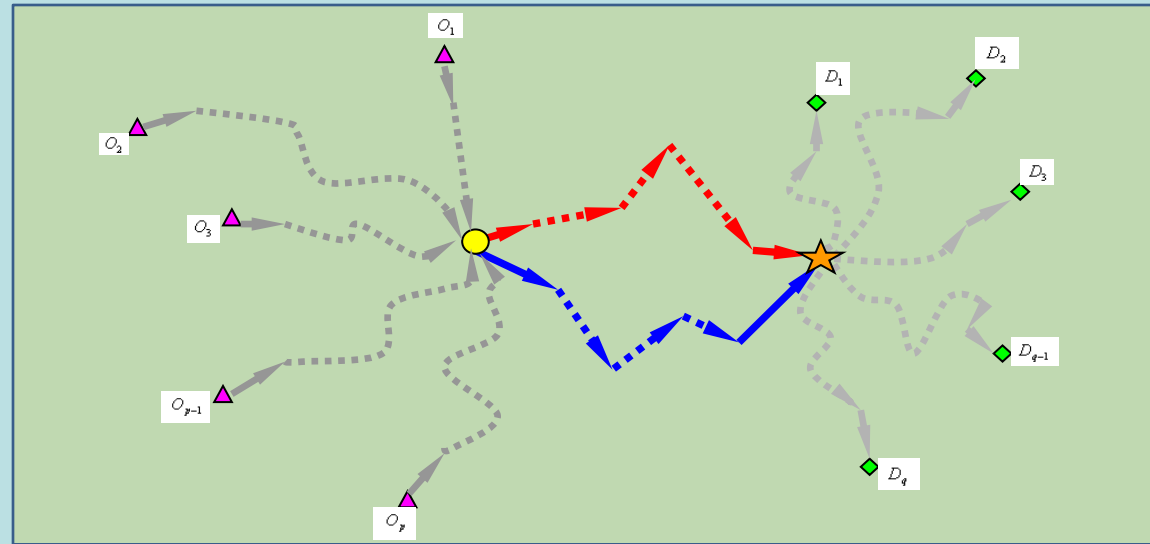
2) Proportionality Adjustment

→ Flow distribution between origins

→ Practically equivalent to MEUE route flow solutions

→ Route flow uniqueness  
(i.e. consistent choices of route flow solutions)

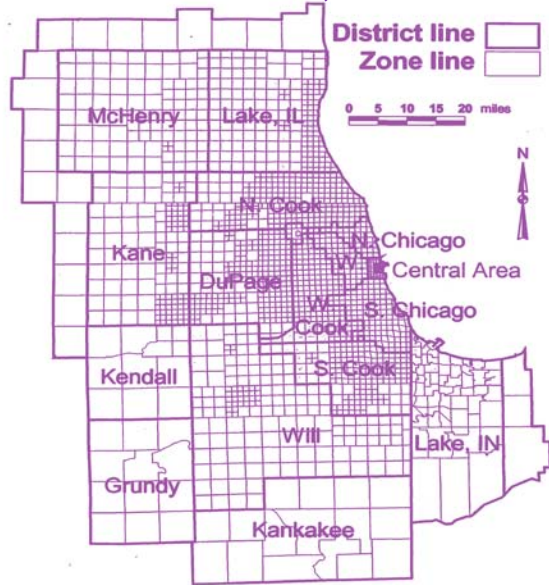
# Fundamental principles of PAS



- Consider only **Pairs of (distinct) Alternative Segments (PAS)**.
- Every PAS consists of two sequences of links connecting a pair of nodes, each called **“the distinguishing component or (alternative) segment”**.
- Every PAS has a **set of relevant origin(s)**, or a **set of relevant destination(s)**.
- Every PAS has **one** originating node, the **diverge node**, and **one** terminating node, the **merge node**.
- A PAS can take **any shape** as long as the segment travel times are precisely equal.
- For every PAS, the routes taken from any origin to a diverge node and from a merge node to any destination **are not at issue**.
- Any PAS can be the **distinguishing component** for many pairs of routes.
- Equilibrium routes between OD pairs may have no or one or more PAS(s).

# Basic characteristics of the Chicago regional network

17 Districts / 1,790 Zones



Physical Characteristics  
of Chicago Regional Zone System

## Mode-Origin-Destination Model:

$$d_{mpq} = A_p B_q \exp(-\mu * c_{mpq})$$

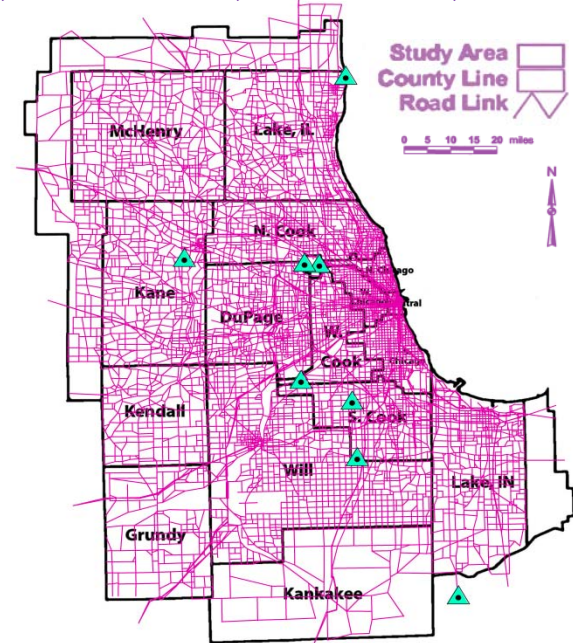
Where  $A_p, B_q$  = Balancing Factor

$\mu$  = Cost Sensitivity Parameters

$c_{mpq}$  = Mode-Origin-Destination

Level of Service/Cost

12,982 Nodes/ 39,018 Links/ 27,050.2 mi

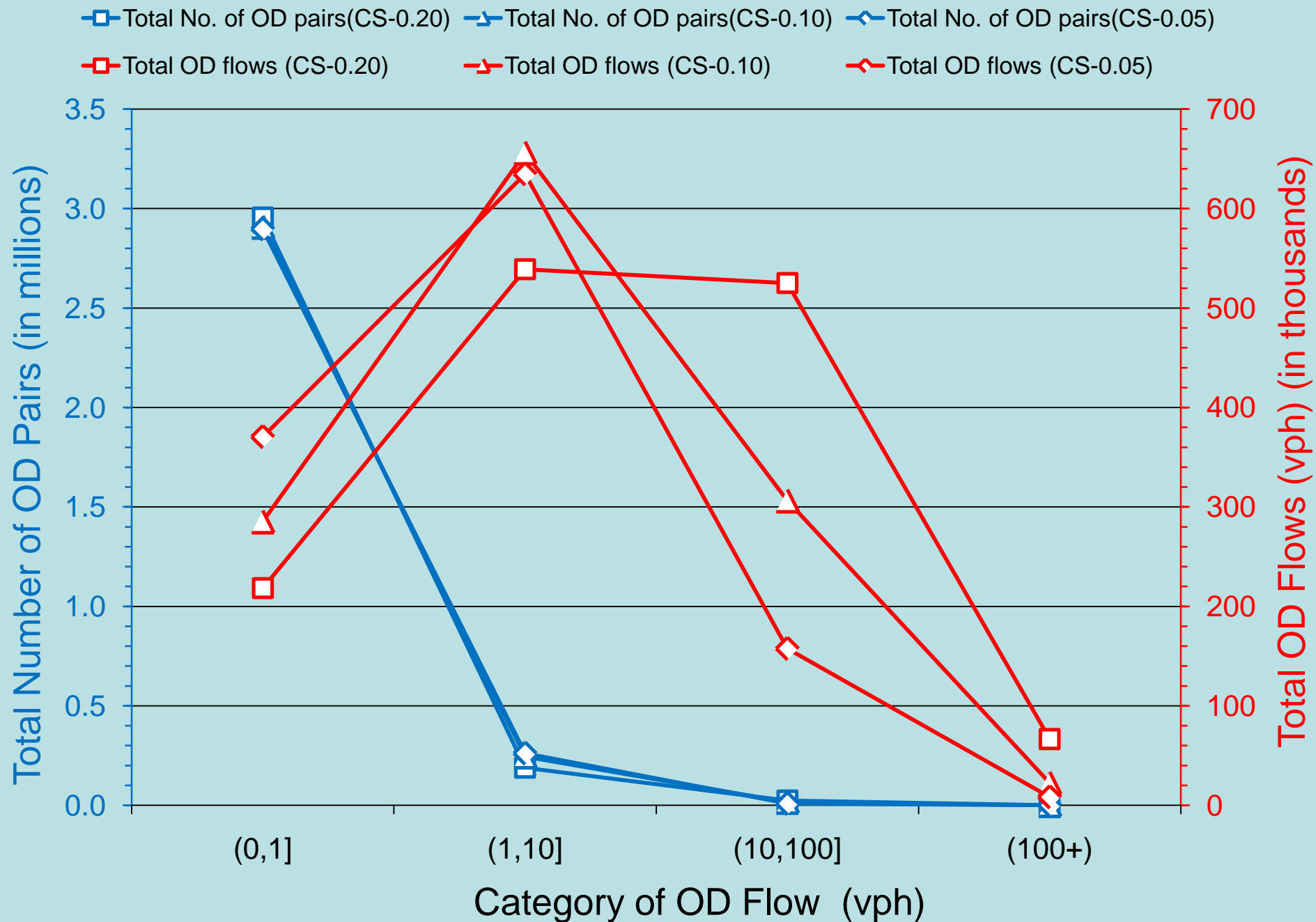


Physical Characteristics  
of Chicago Regional Road Network

## General characteristics of three OD trip matrices

Cost Sensitivity	Total number of O/D Zones	Number of OD pairs				Total number of OD pairs	OD flows		Total OD flows
		Interzonal		Intrazonal			Interzonal	Intrazonal	
		with flow	without flow	with flow	without flow				
0.20	1,790	3,168,206 (98.9%)	34,104 (1.0%)	1,771 (0.1%)	19 (0.0%)	3,204,100 (100%)	1,349,081.1 (94.4%)	80,820.0 (5.6%)	1,429,901.2 (100%)
0.10	1,790	3,168,206 (98.9%)	34,104 (1.0%)	1,771 (0.1%)	19 (0.0%)	3,204,100 (100%)	1,269,957.0 (95.9%)	54,916.8 (4.1%)	1,324,873.8 (100%)
0.05	1,790	3,168,206 (98.9%)	34,104 (1.0%)	1,771 (0.1%)	19 (0.0%)	3,204,100 (100%)	1,171,166.1 (96.6%)	41,884.9 (3.4%)	1,213,051.0 (100%)

# Distribution of Interzonal OD Flows

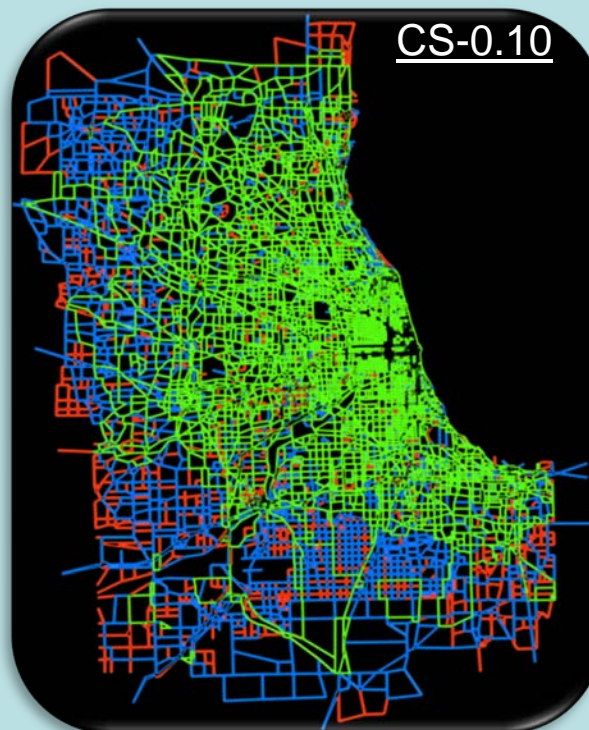


# Aggregated characteristics of PAS solutions-1

A RED link is an UNUSED link.

A GREEN link is a USED link that is a PART of a PAS.

A BLUE link is a USED link that is NOT PART of any PAS.



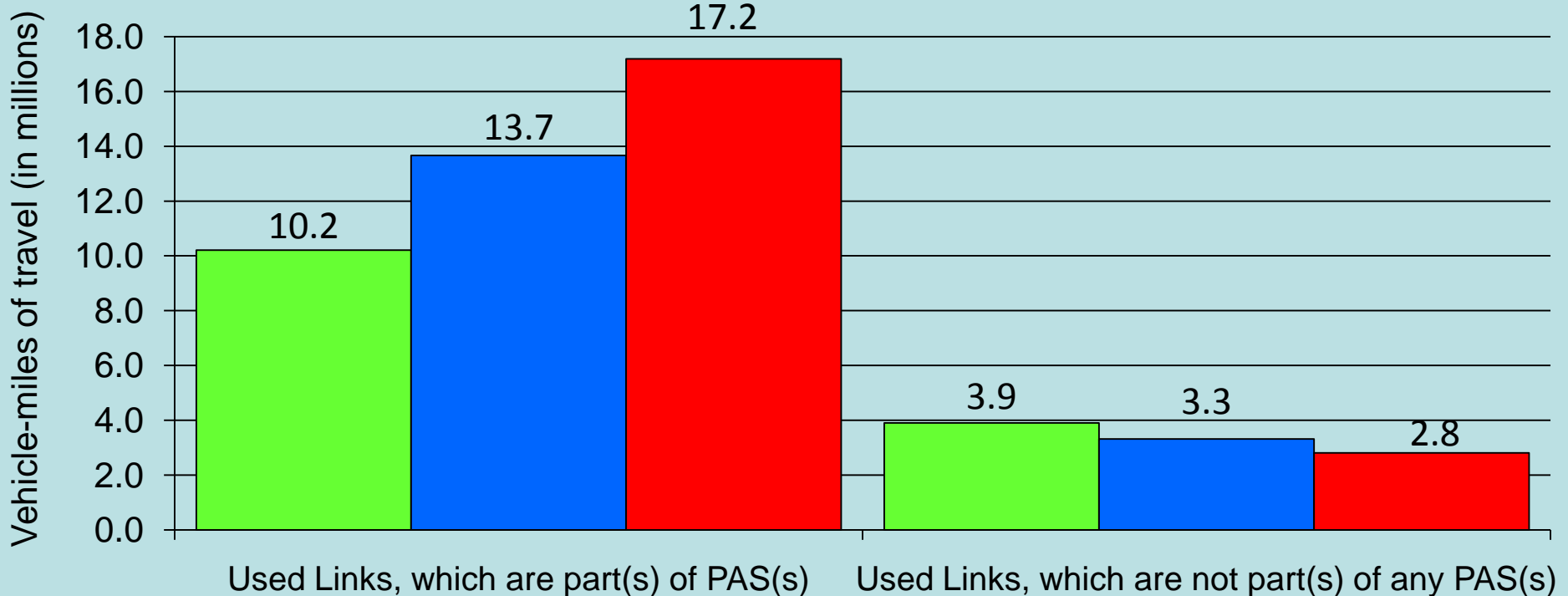
# Aggregated characteristics of PAS solutions-2

Cost sensitivity	Mean route travel time (minutes)	Number of PASs	Total length of PASs (miles)		PAS coverage (%) (a)	Length per PAS (miles)	Number of links per PAS	Number of origins per PAS	Number of equilibrium routes
			Overlaps included	Overlaps excluded					
0.20	52.6	5,617	57,342	8,7634	32.4	10.2	19.9	105.2	8,397,772
0.10	60.3	11,702	153,857	11,403	42.2	13.2	27.1	100.6	19,121,834
0.05	80.1	22,500	407,264	13,693	50.6	18.1	35.5	103.3	198,087,738

(a) (total length of all non-overlapping links on all PASs)/(total length of all links over a network)x100

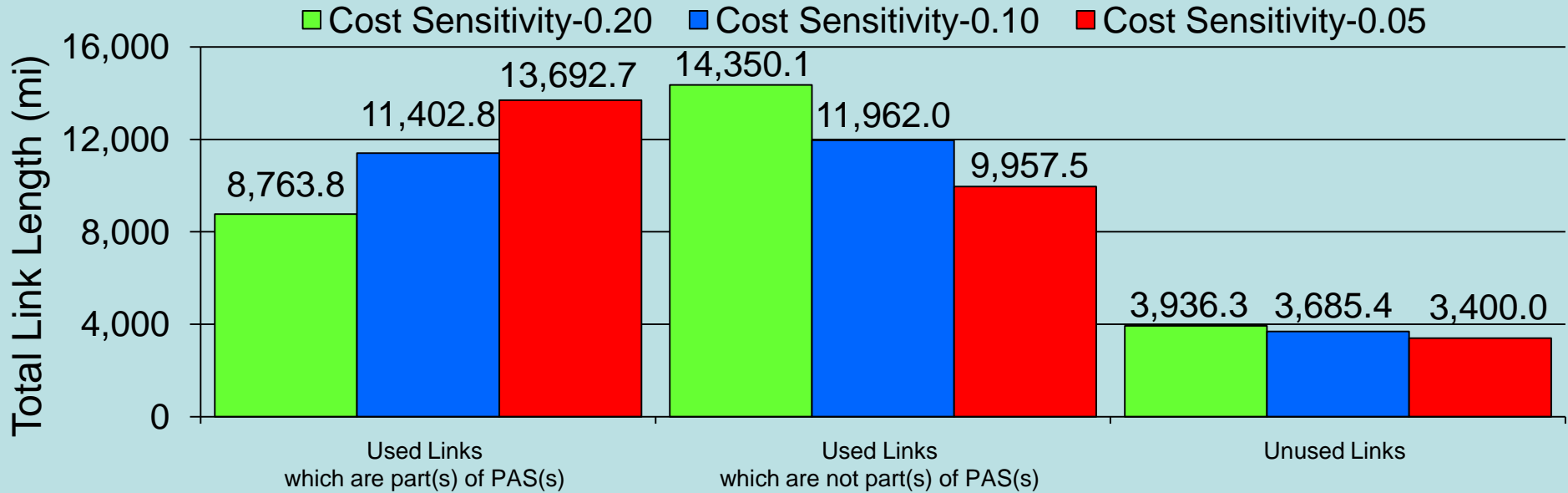
## Distribution of Vehicle-Miles of Travel of PAS solutions

■ Cost Sensitivity-0.20 ■ Cost Sensitivity-0.10 ■ Cost Sensitivity-0.05

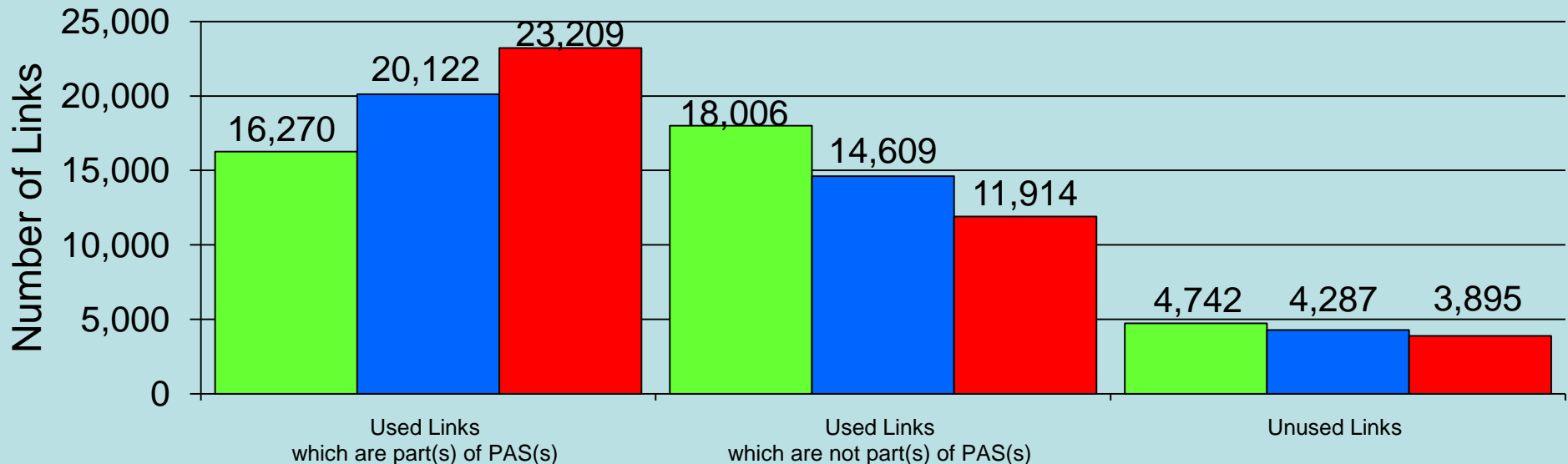


# Aggregated characteristics of PAS solutions-3

## Distribution of Link Length of PAS solutions



## Distribution of Number of Links of PAS solutions

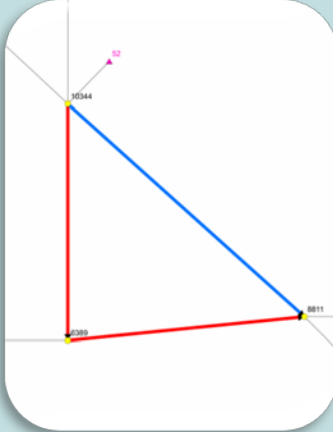


# Disaggregated characteristics of PAS solutions - 1

- All PASs must perfectly conform to the PAS definition.
- No alternative segment is crossed at a node.

## PASs without segment CROSSING

Simple PAS



Simple PAS



Complex PAS



Complex PAS



## PASs with segment CROSSING(S)

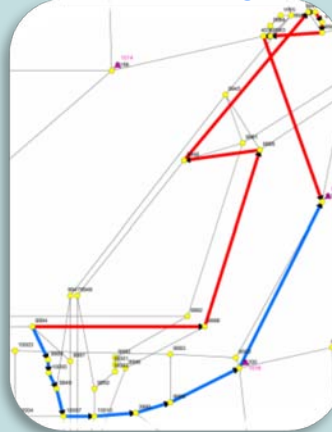
1 crossing



2 crossings



2 crossings



3 crossings

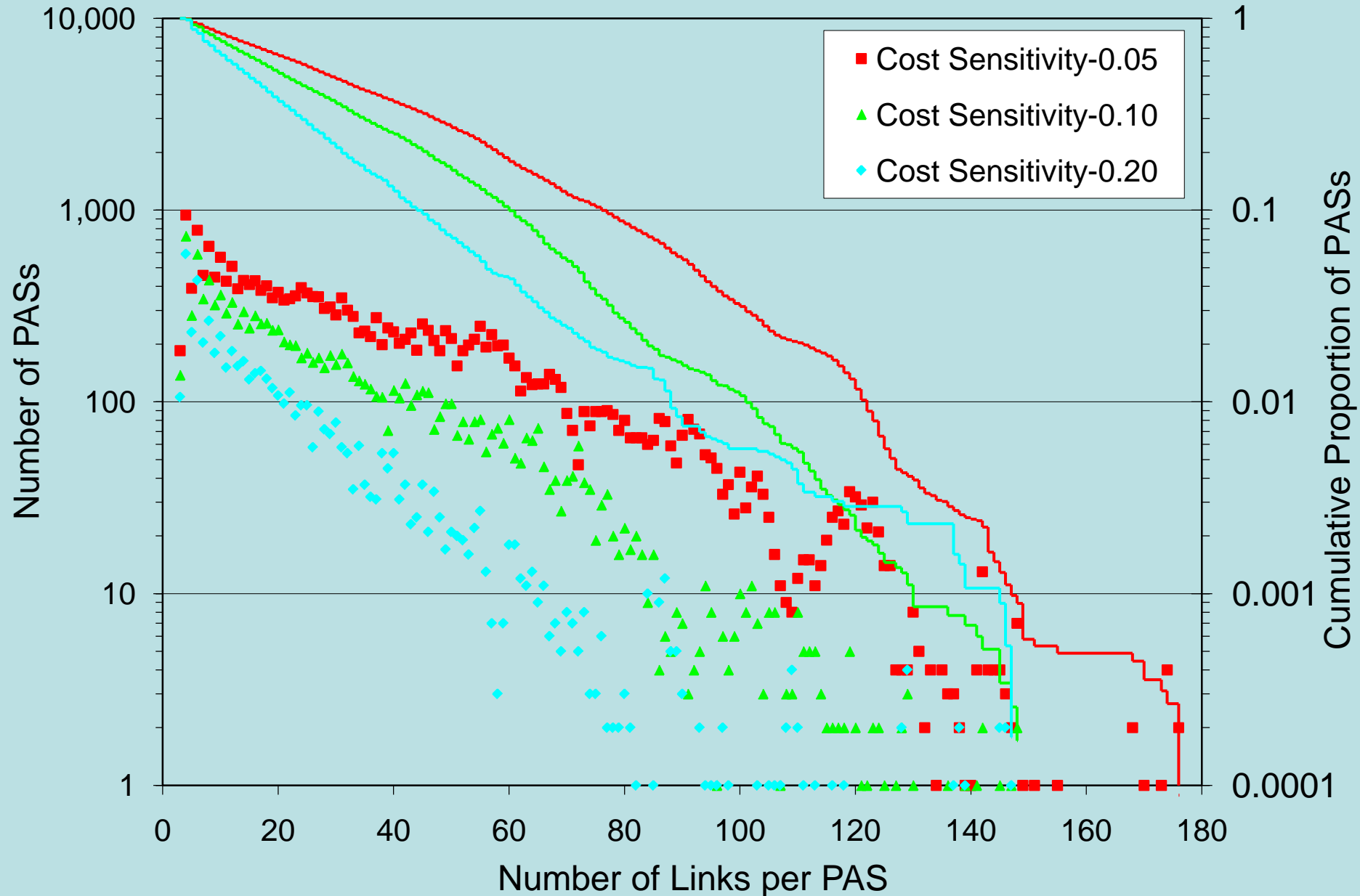


3+ crossings



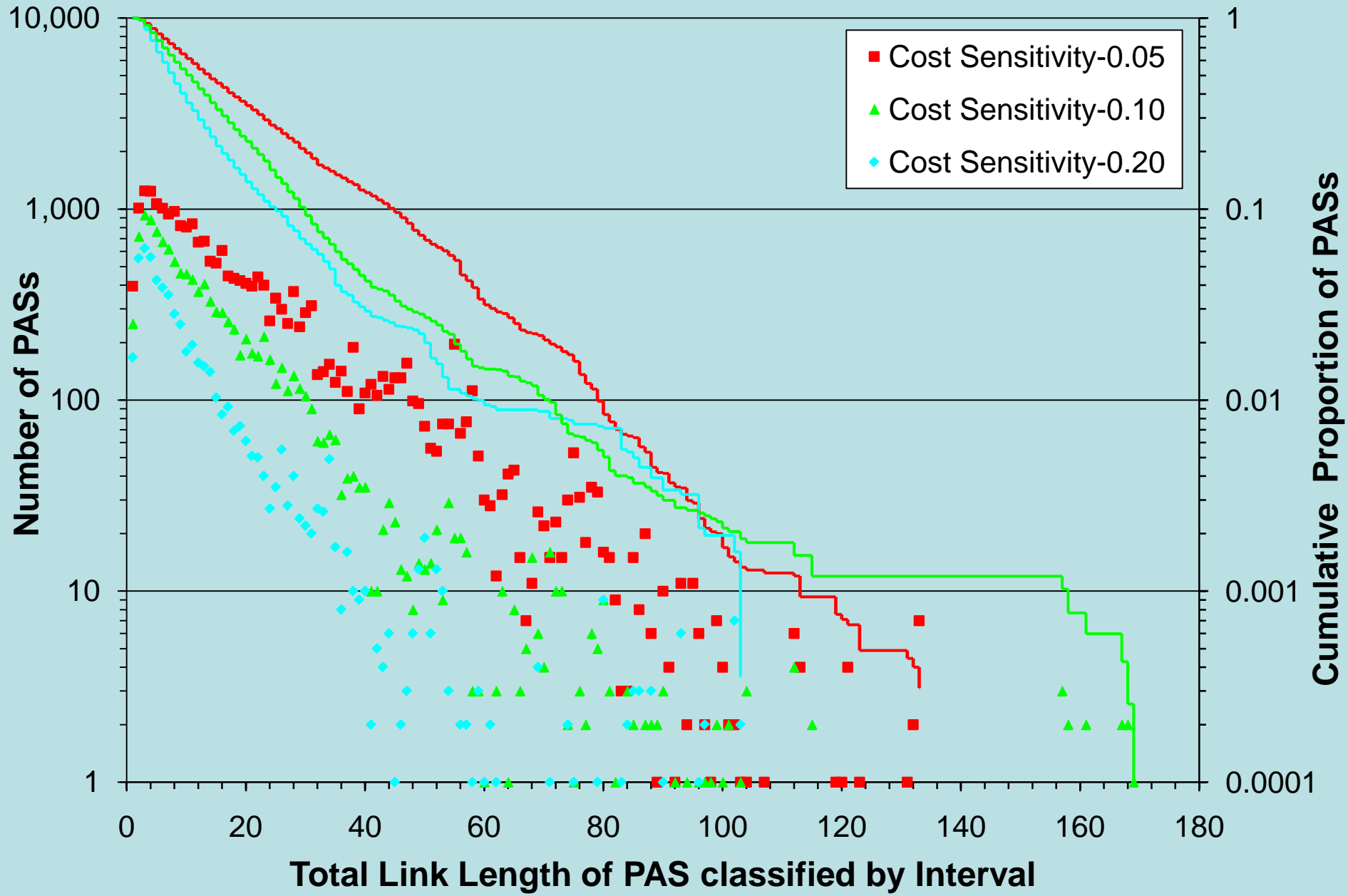
# Disaggregated characteristics of PAS solutions - 2

## Number of PASs versus Number of Links per PAS



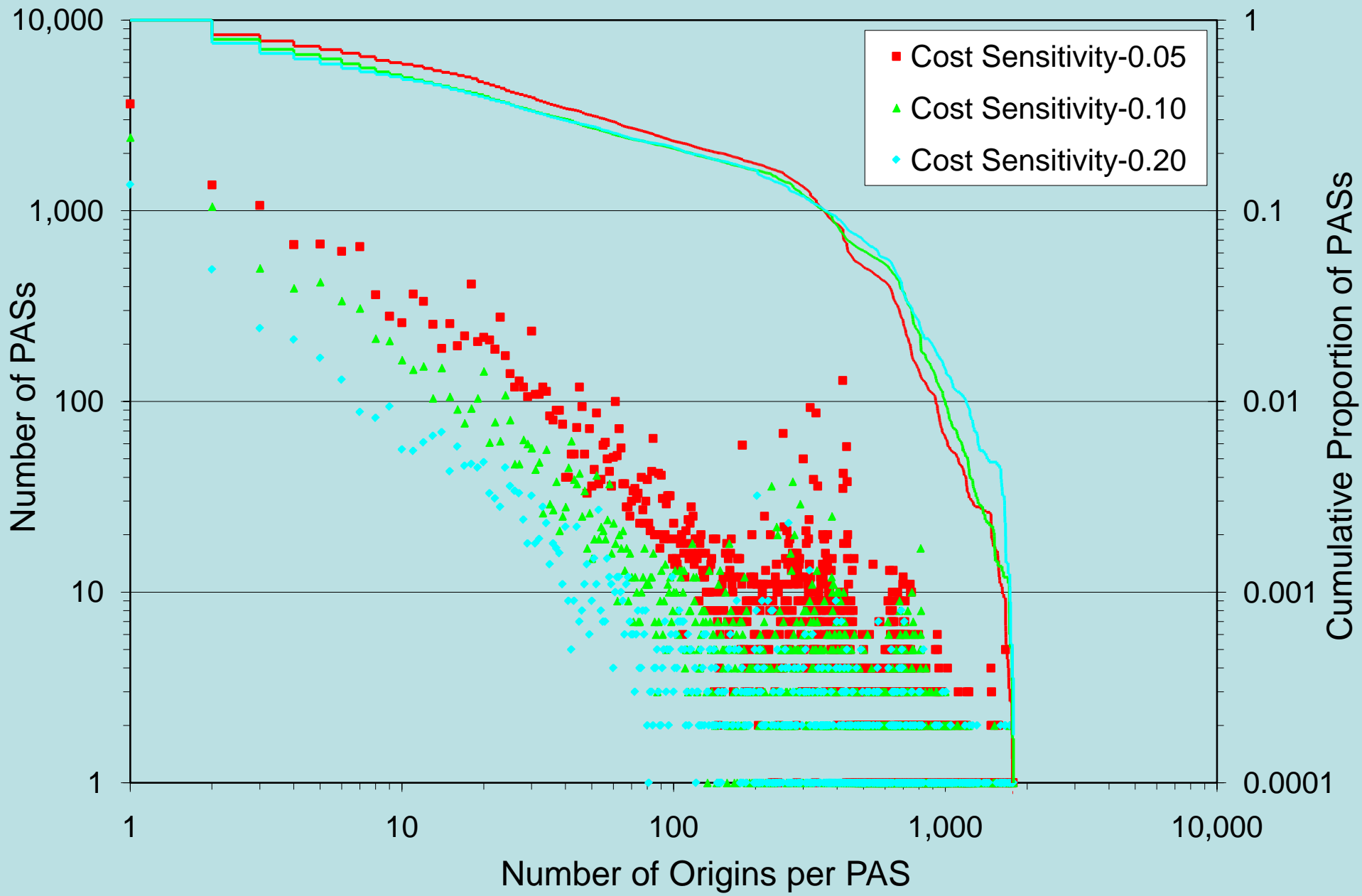
# Disaggregated characteristics of PAS solutions - 3

Number of PASs versus Total Link Length of PAS classified by Interval



# Disaggregate characteristics of PAS solutions - 4

## Number of PASs versus Number of Origins per PAS

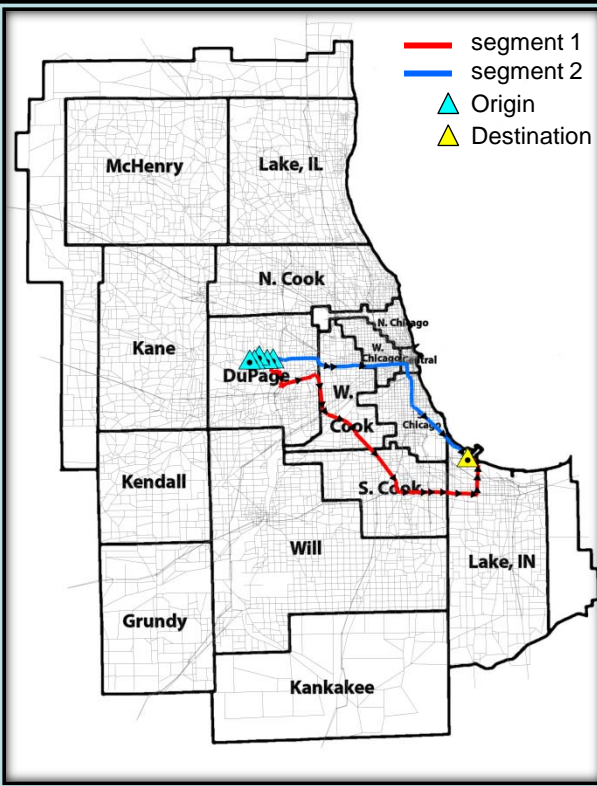


# Disaggregate characteristics of PAS solutions - 5

## Location and characteristics of PASs with maximum length by trip matrix

### CS-0.20

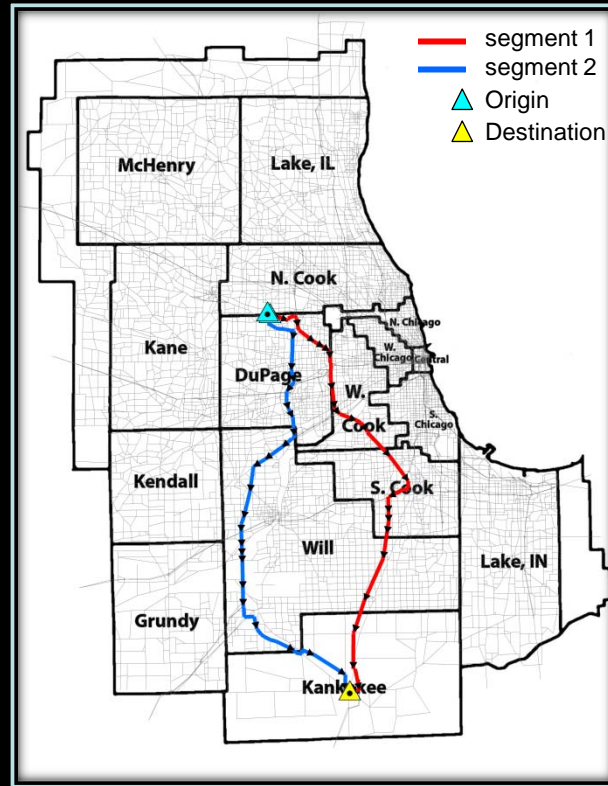
Length1: 58.8    Cost: 71.01  
Length2: 43.5    Flow1: 0.024  
#Links: 145    Flow2: 0.073  
#Origins: 5    Prop1: 0.249  
#Destinations: 1    Prop2: 0.751



Total Link Length of PAS =102.3 mi

### CS-0.10

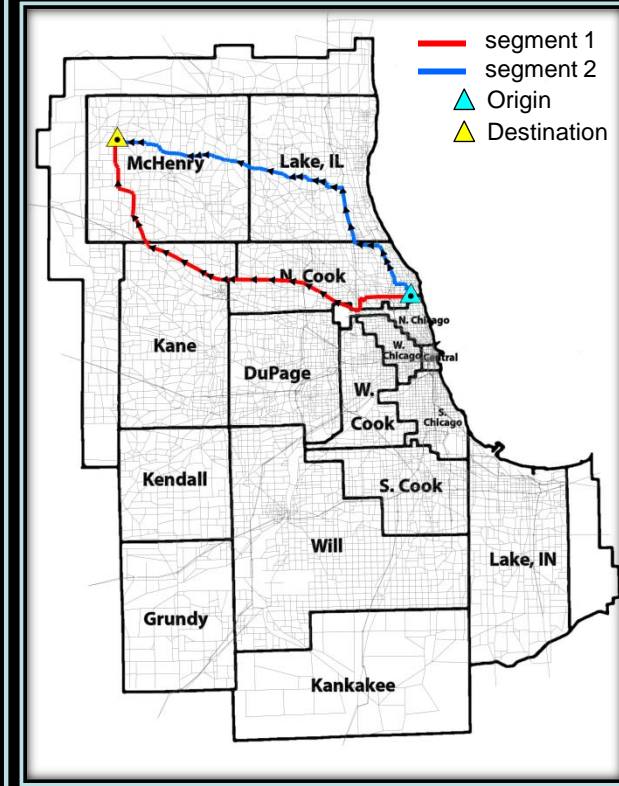
Length1: 85.2    Cost: 97.14  
Length2: 83.0    Flow1: 0.113  
#Links: 120    Flow2: 0.003  
#Origins: 2    Prop1: 0.975  
#Destinations: 1    Prop2: 0.025



Total Link Length of PAS =168.2 mi

### CS-0.05

Length1: 68.7    Cost: 96.95  
Length2: 64.1    Flow1: 0.00001  
#Links: 174    Flow2: 0.00144  
#Origins: 1    Prop1: 0.007  
#Destinations: 1    Prop2: 0.993



Total Link Length of PAS =132.9 mi

# Contributions of PASs to formation of equilibrium route patterns

## (Aggregation by an Origin)

### Origins with Maximum Number of PASs on **CS-0.20**

#### CS-0.20

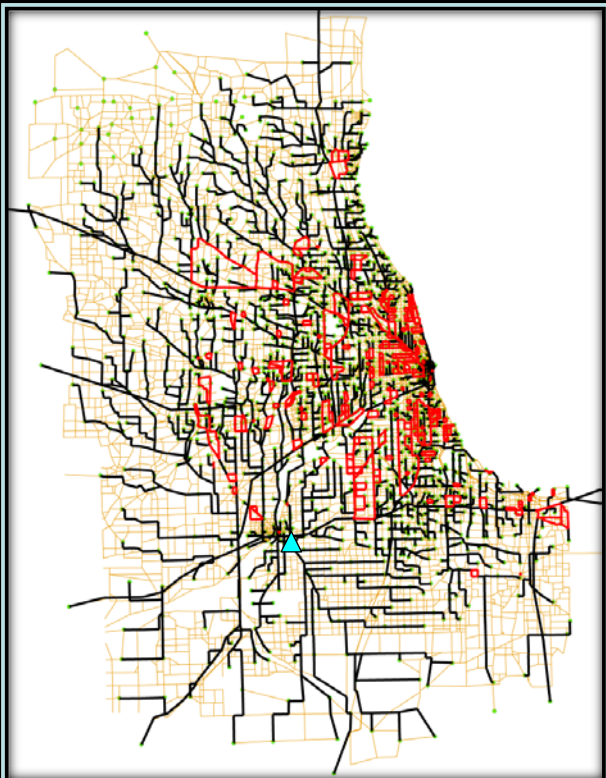
512 PASs            130.67 vph  
3,378 routes(total)    55.79 min  
855 routes(single)

#### CS-0.10

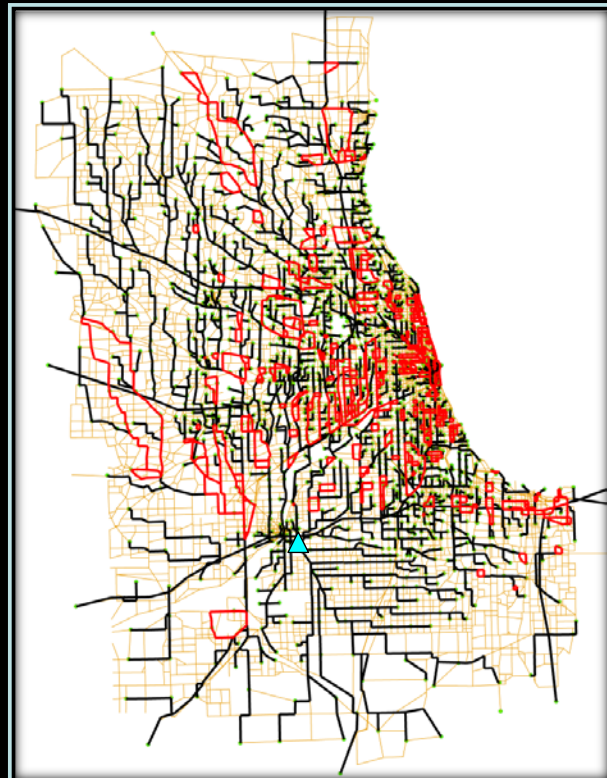
945 PASs            120.71 vph  
6,687 routes(total)    66.26 min  
576 routes(single)

#### CS-0.05

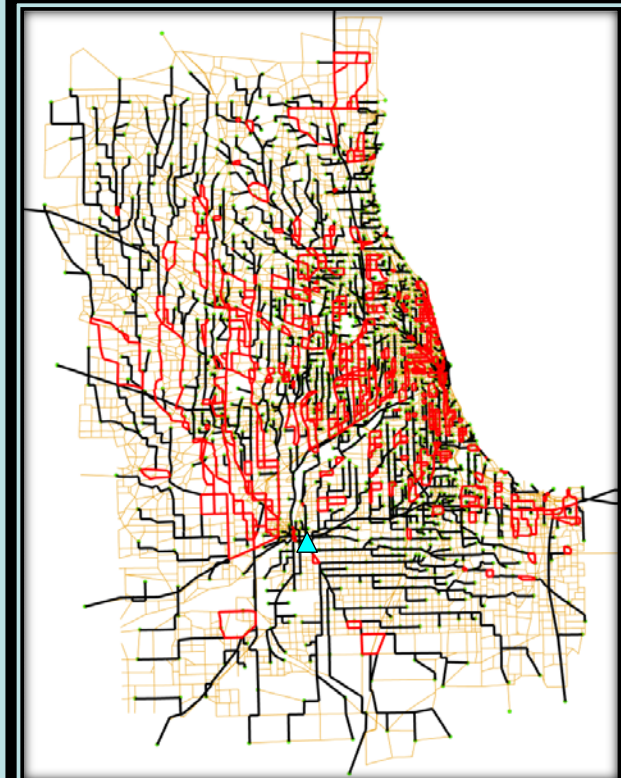
1,574 PASs            102.10 vph  
51,847 routes(total)    81.39 min  
445 routes(single)



from Origin 1608



from Origin 1608



from Origin 1608

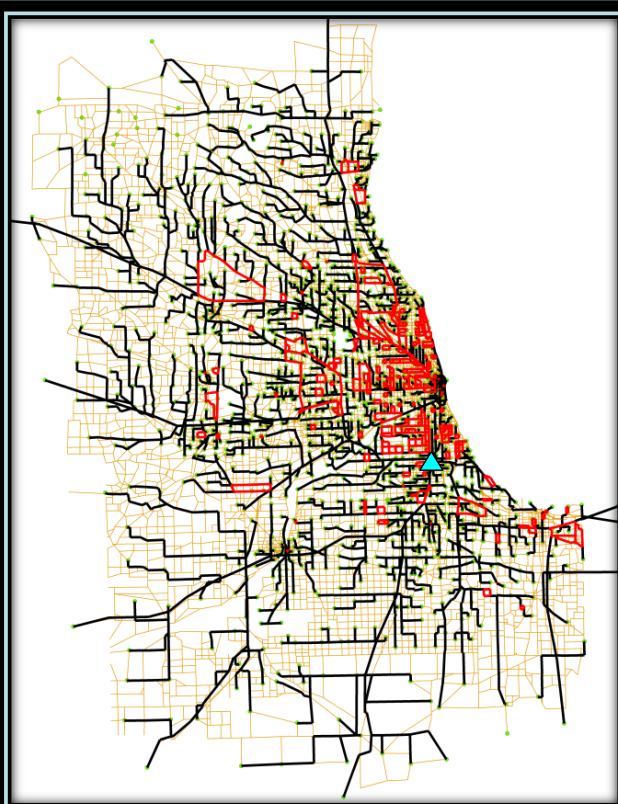
# Contributions of PASs to formation of equilibrium route patterns

(Aggregation by an Origin)

Origins with Maximum Number of PASs on **CS-0.10**

CS-0.20

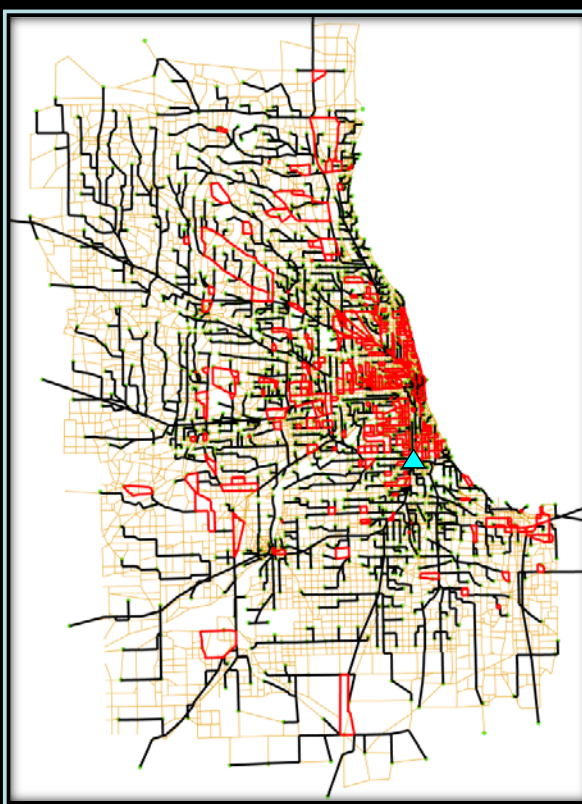
478 PASs      822.70 vph  
2,604 routes(total)    39.92 min  
1,459 routes(single)



from Origin 183

CS-0.10

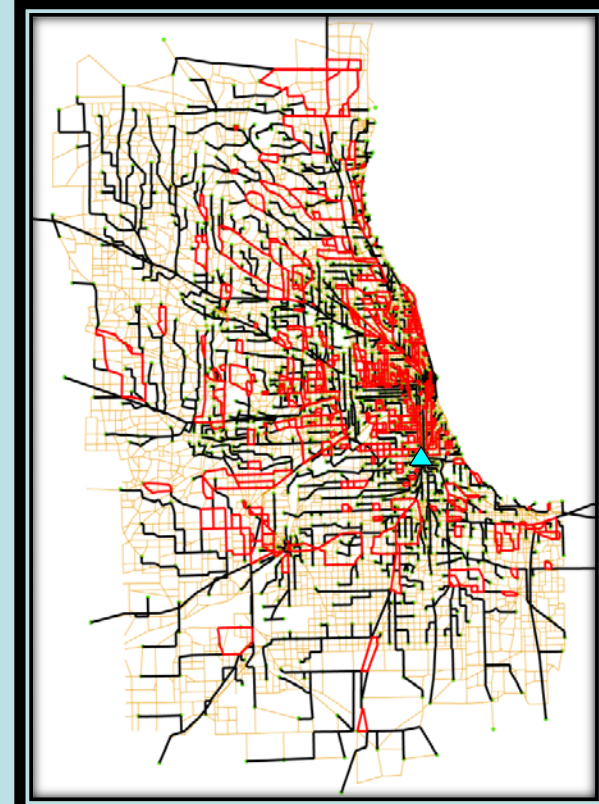
1,058 PASs      851.59 vph  
6,293 routes(total)    46.19 min  
887 routes(single)



from Origin 183

CS-0.05

2,034 PASs      826.56 vph  
180,227 routes(total)    64.86 min  
363 routes(single)



from Origin 183

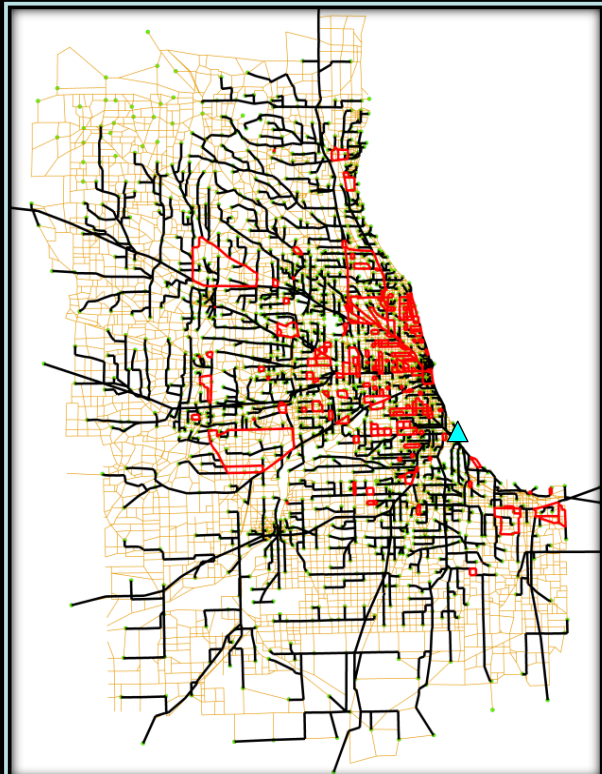
# Contributions of PASs in formulation of equilibrium route patterns

(Aggregation by an Origin)

Origins with Maximum Number of PASs on **CS-0.05**

CS-0.20

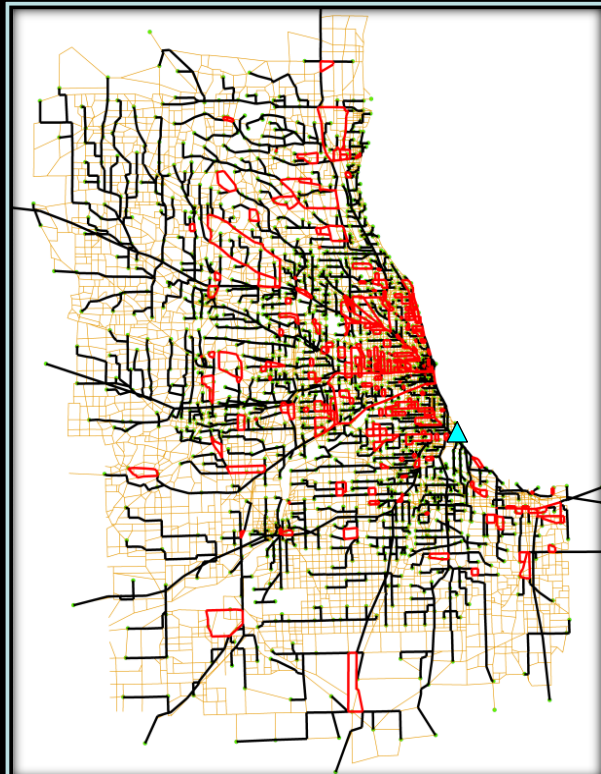
409 PASs      160.52 vph  
2,970 routes(total)    44.47 min  
1,111 routes(single)



from Origin 3

CS-0.10

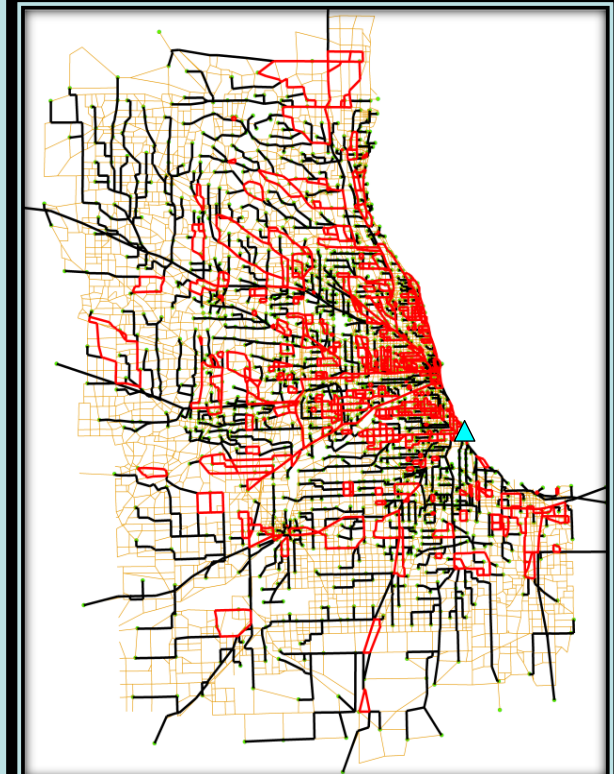
889 PASs      164.84 vph  
7,019 routes(total)    48.32 min  
1,035 routes(single)



from Origin 3

CS-0.05

2,472 PASs      169.54 vph  
792,630 routes(total)    62.26 min  
270 routes(single)



from Origin 3

# Contributions of PASs in formulation of equilibrium route patterns

(Disaggregation by an OD pair)

OD Pairs with Maximum Total Link Length of PAS on **CS-0.20**

CS-0.20

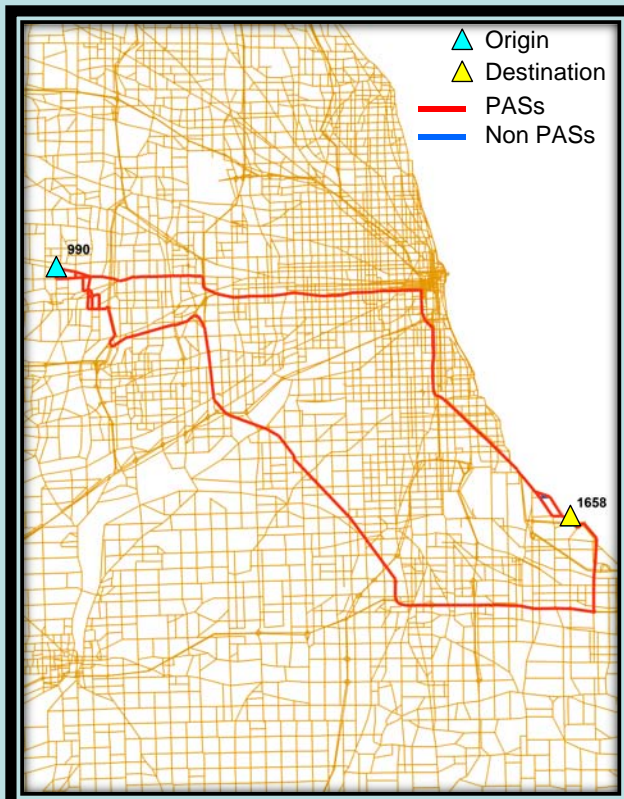
21 PASs      86.45 min  
42 routes      65.98 mi  
0.140027 vph      45.86 mph

CS-0.10

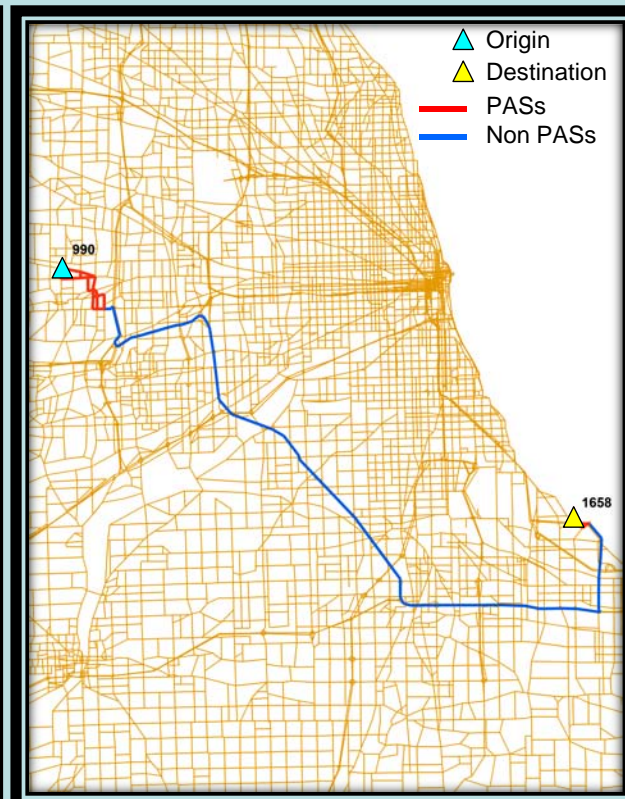
17 PASs      89.53 min  
50 routes      62.63 mi  
0.150210 vph      41.97 mph

CS-0.05

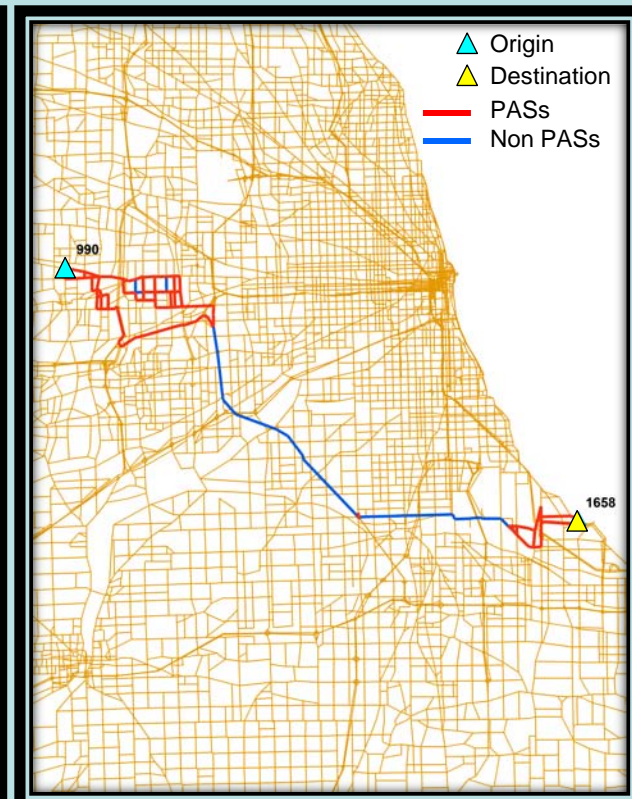
55 PASs      111.59 min  
294 routes      51.08 mi  
0.283558 vph      27.46 mph



OD Pair 990 - 1658



OD Pair 990 - 1658



OD Pair 990 - 1658

# Contributions of PASs in formulation of equilibrium route patterns

(Disaggregation by an OD pair)

OD Pairs with Maximum Total Link Length of PAS on **CS-0.10**

CS-0.20

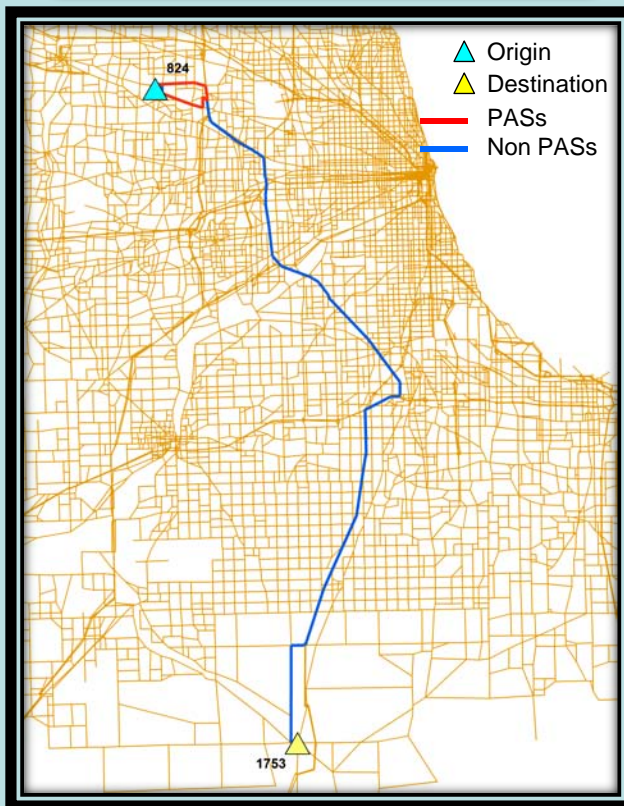
1 PAS	90.77 min
2 routes	84.73 mi
0.410000 vph	56.00 mph

CS-0.10

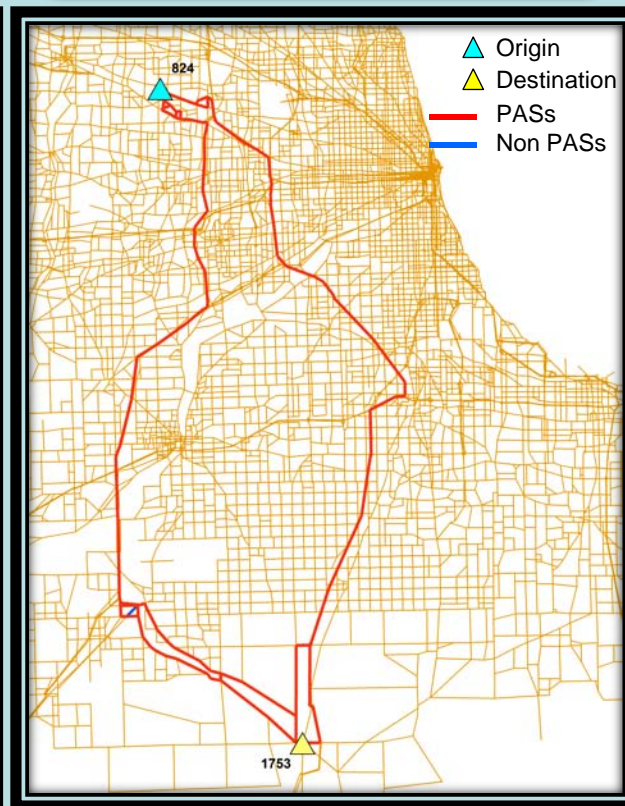
21 PASs	98.04 min
10 routes	85.57 mi
0.411146 vph	52.37 mph

CS-0.05

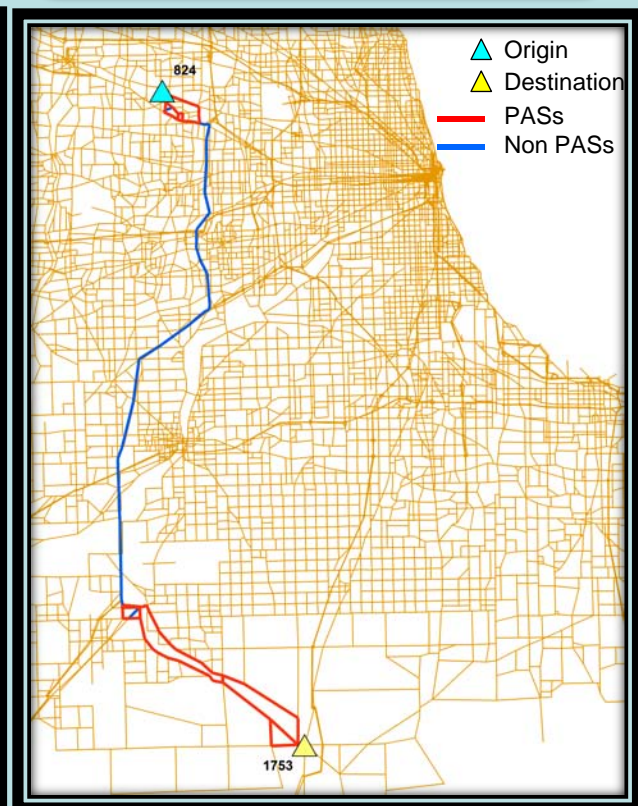
25 PASs	108.56 min
20 routes	85.04 mi
0.495564 vph	47.00 mph



OD Pair 824 - 1753



OD Pair 824 - 1753



OD Pair 824 - 1753

# Contributions of PASs in formulation of equilibrium route patterns

(Disaggregation by an OD pair)

OD Pairs with Maximum Total Link Length of PAS on **CS-0.05**

CS-0.20

0 PAS	80.45 min
1 route	71.71 mi
1.09E-9 vph	53.45 mph

CS-0.10

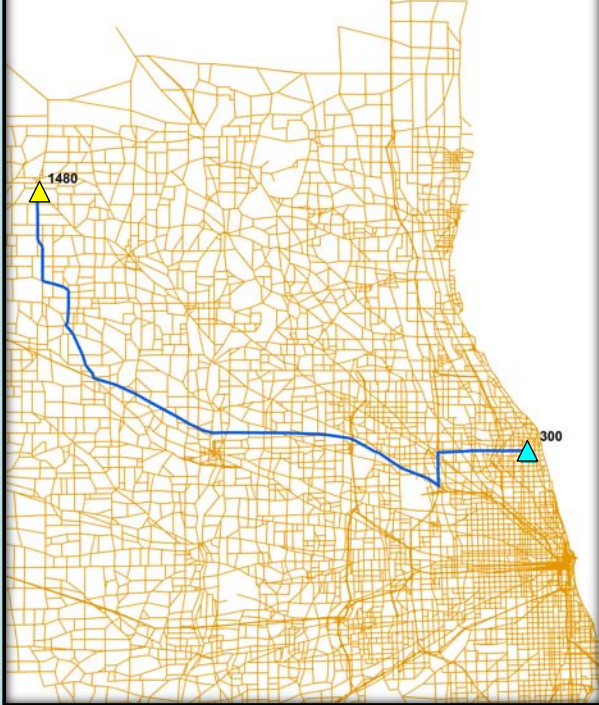
5 PASs	86.45 min
6 routes	65.98 mi
0.000222 vph	45.79 mph

CS-0.05

29 PASs	96.95 min
16 routes	67.99 mi
0.017206 vph	42.08 mph

▲ Origin  
▲ Destination

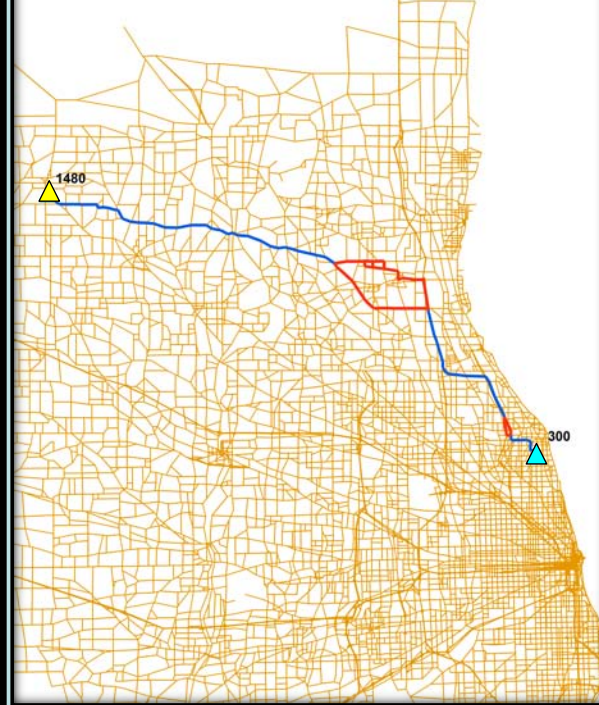
— PASs  
— Non PASs



OD Pair 300 - 1480

▲ Origin  
▲ Destination

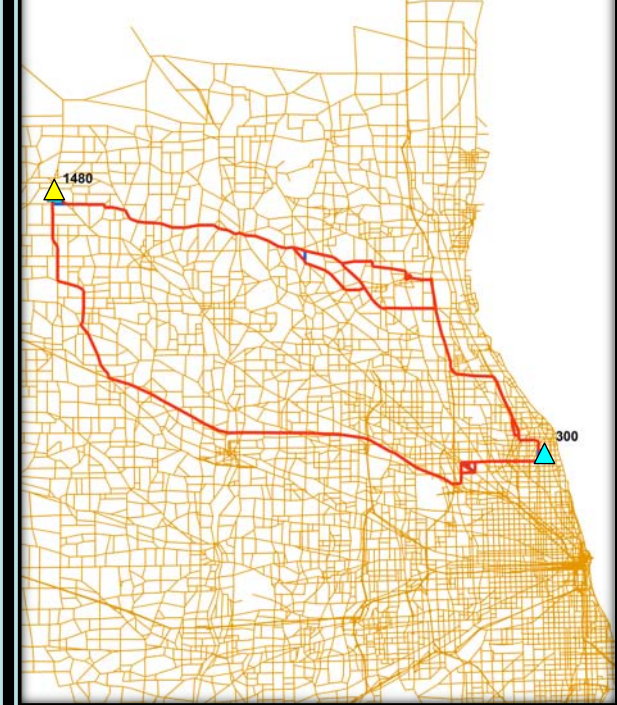
— PASs  
— Non PASs



OD Pair 300 - 1480

▲ Origin  
▲ Destination

— PASs  
— Non PASs



OD Pair 300 - 1480

## Summary of findings for PAS solutions

The following findings are expected to help guide transportation professionals and practitioners in understanding the properties of PASs for solving traffic assignments with unique route flows:

- No active PASs has a segment crossing at a node.
- All active PASs consist of physically unique links.
- Simple PASs formed by three or four links are prevalent.
- Active PASs which are short and local are commonplace.
- Occurrences of PASs which are extremely long are rare.
- Active PASs with small numbers of origins occur most frequently.
- All active PASs relevant to a specific origin are part(s) of a corresponding tree of minimum travel cost routes.

## Questions for future research

This research seeks to understand characteristics of user equilibrium solutions to large scale urban traffic networks. Further research is needed on:

- Networks from other regions
- Assignment of multiple classes
- Different trip matrices
- More complete generalized link cost functions including distance term and tolls